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On the absolute instability of the triple–deck flow over humps and near wedged trailing edges

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THE ROYAL SOCIETY

On the absolute instability of the triple-deck flow over humps and near wedged trailing edges

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The triple-deck equations for the flow over a hump, a corner and a wedged trailing edge are solved numerically using a novel method based on spectral collocation. It is found that for the flow over a corner, separation begins at a scaled angle β of 2.09, and for the wedged trailing edge for a wedge angle of 2.56. Here β is defined in terms of the small physical angle ϕ by $\beta = Re^{1/4}\lambda^{-1/2}\phi$, $\lambda = 0.3320$, and Re is the Reynolds number. The absolute instability of the nonlinear mean flows computed is investigated. It is found that the flow over a hump is inviscidly absolutely unstable with the maximum absolute unstable growth rate occurring near the maximum height of the hump, and increasing with hump size. The wake region behind the wedged trailing edge is also found to be absolutely unstable beyond a critical wedge angle, and the extent of the region of absolute instability increases with increasing wedge angle and separation.

Keywords: boundary layer; separation; stability; triple deck

1. Introduction

It has been known since Goldstein's (1930) paper that classical boundary-layer theory breaks down near the trailing edge of a finite flat plate, owing to the presence of a large induced pressure gradient there. In order to properly account for this singular behaviour and provide continuation of the Blasius solution into the wake, new scalings need to be introduced. Stewartson (1969, 1970), Neiland (1969) and Messiter (1970) have derived a rational asymptotic expansion of the flow variables near the trailing edge, and the resulting disturbance structure that they discovered is known as the triple-deck structure. The same structure arises in many other related contexts including near the separation point in an adverse pressure gradient boundary layer, in shock-wave boundary-layer interactions, and in the stability of boundary layers. A comprehensive account of the origins and applications of triple-deck theory may be found in Stewartson (1974, 1981), Smith (1982) and Sychev *et al.* (1998).

Our concern in this paper is primarily with the flow near the trailing edge of a wedge-shaped aerofoil. Various numerical methods have been developed to solve the equations governing triple-deck viscous–inviscid interaction problems. For the trailing-edge flow, the presence of an abrupt discontinuity in the boundary conditions at the trailing edge, and also the interaction law relating the pressure and the displacement thickness of the boundary layer for subsonic flow, makes the numerical treatment of the equations considerably difficult. Daniels (1974*a*, *b*) solved the equations using a numerical marching procedure to calculate the symmetric flow near the trailing edge of a flat plate at zero incidence and the asymmetric supersonic flow over a flat plate at an angle of attack. He also computed asymptotic solutions for near-trailing-edge and downstream flow. The first numerical solution of the interaction problem for subsonic flow near the trailing edge of a flat plate was given by Jobe & Burggraf (1974). They proposed a combined inverse method in which, for a given displacement distribution, the pressure was computed and the displacement updated using this new pressure. Their calculations showed that an acceleration of the fluid ahead of the trailing edge of the plate is always present due to the singular behaviour of an adverse pressure gradient. Chow & Melnik (1976) applied this method for calculating the asymmetric flow over the trailing edge of a flat plate at an angle of attack. Smith (1974) developed a two-region matching procedure that very closely follows the mathematical development of the double-layered solution near the singularity. Making use of this method, he solved the problem of slot injection into the fluid from a flat plate. Daniels (1974*a*, *b*) exploited this technique for trailing-edge flows.

The numerical solution of the nonlinear interaction problem was also obtained by Ruban (1976, 1977). His numerical method was based on solving the boundary-layer equations for a given displacement thickness and combined with the method of Jobe & Burggraf (1974). Using this method, Ruban (1976, 1977) calculated separation occurring around a surface irregularity and near the trailing edge of a thin symmetric aerofoil. Later, both Ruban & Sychev (1979) and Smith & Merkin (1982) investigated the triple-deck flow around a wedge-shaped trailing edge. The latter also extended their studies to the viscous–inviscid interaction due to a small hump and convex/concave corners. They transformed the infinite physical domain completely into a finite range of streamwise integration, and simple transformations were made for the pressure and displacement function to prevent the original unbounded growth of these two functions. They investigated separation taking place in an incompressible flow near a corner of a body, near the wedge-shaped trailing edge of a thin aerofoil, and some other external flow structures. Their results predicted the scaled angles β, 2.51, -5.21 and 2.38, for the onset of the separation at a concave corner, a convex corner, and a wedged trailing edge, respectively. The first and last values are somewhat different from the corresponding values of 2.0 and 2.6 that were computed by Ruban (1976, 1977) and also given in Sychev *et al.* (1998).

The calculations performed by Ruban (1976, 1977), Ruban & Sychev (1979) and Smith & Merkin (1982) were based on the use of iterative schemes which became divergent when the region of recirculating flow was sufficiently large. To study the behaviour of the solution at larger values of β for the corner flow, Korolev (1991, 1992) employed a direct method and calculated a recirculating zone up to $\beta = 7$. A further increase in β created a singularity in the skin friction immediately ahead of the attachment point, and thus made the use of interaction theory invalid.

Also of great interest is the stability of the locally distorted steady or unsteady separated flow motions. In addition to the Tollmien–Schlichting modes of instability, the triple-deck solutions admit inflectional velocity profiles which are susceptible to an inviscid Rayleigh-type instability. Rayleigh instability has typical length-scale much shorter than that of Tollmien–Schlichting instability and represents a bursting phenomenon due to the fairly sudden production of faster spatial and temporal growth effects. Smith & Bodonyi (1985) have shown that the triple-deck solutions over a mounted-surface obstacle are subjected to a short-scale Rayleigh-type instability, provided that the obstacle size is sufficiently large to generate nonlinear solutions. Using temporal stability theory, they have computed the temporal growth rates at different locations corresponding to flow-reversal regions. Duck (1985, 1988) has also encountered an inflectional instability in unsteady incompressible boundary-layer flow computations, which manifested itself in such a way that large wavenumbers were excited by surface distortions, leading to the breakdown of the solution as a finite time is approached.

The present work is concerned with the triple-deck flows over humps, a concave corner, and wedged trailing-edge configurations. There are several main objectives. The first is to develop a viscous–inviscid interaction code using spectral methods, which have the distinct advantage that greater accuracy is easily obtained with only a modest increase in the number of points used. Another aim is to compute solutions for the wedged trailing-edge flows with the scaled angle parameter β larger than those previously computed. Another aspect of the current work is to resolve the controversy in the difference, as far as the critical angle at the onset of separation is concerned, between the two sets of results given by Ruban (1976, 1977) and Smith & Merkin (1982). Finally, the work of Smith & Bodonyi (1985) suggests the occurrence of Rayleigh instability for some classes of nonlinear triple-deck mean flows. It is well known that many profiles involving backflow and separation are prone to absolute instability (see Huerre & Monkewitz 1990; Gaster 1984). Another objective is, thus, to examine the inviscid stability of some of these mean flows and to investigate whether these flows are absolutely unstable or not.

In § 2, the triple-deck equations governing the flow in the vicinity of the trailing edge are given and brief details of the numerical method are outlined. Our mean-flow calculation results are presented in § 3. Some stability results are given in § 4. Finally, a summary and conclusions are given in § 5.

2. Problem formulation and solution of the triple-deck equations

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Triple-deck theory divides the main region of the flow into three subregions, namely the lower, main and upper decks. The governing equations in each of these regions are given in Smith (1982) and Sychev *et al.* (1998). In order to construct uniformly valid composite solutions, which are used for the stability computations, we will be interested in the expansions in the main and lower decks only. Note that the contribution to the composite solution from the upper deck will be disregarded, because the effects there are small, $O(Re^{-1/4})$, where Re denotes the Reynolds number based on the chord-wise extent of the aerofoil.

Consider now a thin symmetric aerofoil, with a sharp wedge-shaped trailing edge, placed in an incompressible fluid flow.

We assume that the undisturbed freestream uniform flow is parallel to the aerofoil's chord-line. Later we select an orthogonal curvilinear coordinate system aligned with the body surface such that the wake centreline is in the streamwise direction. We confine ourselves to the consideration of only the upper plane, i.e. $y \geqslant 0$, taking into account the symmetry.

The two regions—namely main and lower decks—are depicted in figure 1 for a wedged trailing edge having a typical wedge angle ϕ. The problem of finding the asymptotic solution to the Navier–Stokes equations for the specific flow is tackled using the well-known triple-deck analysis under the limit $Re \to \infty$. The full analysis

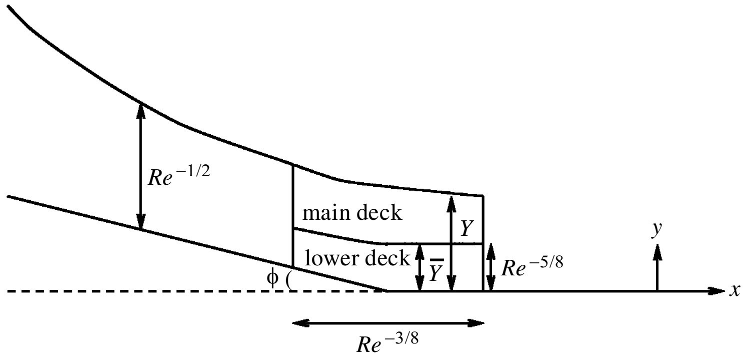

Figure 1. Triple-deck region and scalings are shown in the vicinity of the trailing edge of a wedged shape. ϕ is the wedge angle and is related to scaled wedge angle parameter β by $\beta = \lambda^{-1/2} Re^{1/4} \phi$.

leading to the triple-deck equations governing the subsonic interaction problem may be found in Stewartson (1969, 1974), Smith & Merkin (1982) and Sychev *et al.* (1998).

(*a*) *Main-deck solution*

In this region the boundary-layer coordinate Y is linked to the physical coordinate y by the relation $y = Re^{-1/2}Y$. Since we assume that the aerofoil is very thin, the velocity at the outer edge of the boundary layer is not very different from the velocity of the oncoming flow. Therefore, the leading term of the velocity expansion in this regime coincides with the Blasius solution, denoted by U_{B} below. Without going into much detail, we write the leading- and second-order expansions for the total streamwise velocity distribution only in the form

$$U = U_{\mathrm{B}}(Y) + Re^{-1/8}(A(X) + H(X))U'_{\mathrm{B}}(Y). \tag{2.1}$$

Here, in the second term, $-A(X)$ denotes the local displacement effect of the boundary layer in the viscous–inviscid interaction, and $H(X)$ represents the influence of the profile thickness on the flow in the boundary layer.

(*b*) *Lower-deck solution*

Here, $y = Re^{-5/8}\bar{Y}$ and, applying the Prandtl transposition theorem, the triple-deck equations in terms of the stream function ψ governing the lower-deck reduce to

$$\frac{\partial\psi}{\partial\bar{Y}}\frac{\partial^2\psi}{\partial x\partial\bar{Y}} - \frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial\bar{Y}^2} = -\frac{\partial P}{\partial x} + \frac{\partial^3\psi}{\partial\bar{Y}^3}. \tag{2.2}$$

The boundary conditions for the trailing-edge flow are

$$\psi = \frac{\partial\psi}{\partial\bar{Y}} = 0 \quad \text{at } \bar{Y} = 0, \quad x < 0, \qquad \psi = \frac{\partial^2\psi}{\partial\bar{Y}^2} = 0 \quad \text{at } \bar{Y} = 0, \quad x > 0, \tag{2.3}$$

$$\psi = \tfrac{1}{2}(\bar{Y} + A(x) + H(x))^2 \quad \text{as } \bar{Y} \to \infty, \qquad \psi \to \tfrac{1}{2}\bar{Y}^2 \quad \text{as } x \to -\infty. \tag{2.4}$$

In the above, $x = \lambda^{-5/4}X$ with X being the streamwise triple-deck scale, $\lambda = 0.3320$, and the conditions in equations (2.3)–(2.4) correspond, respectively, to no-slip, wake symmetry, condition of matching, and merging with the Blasius solution. The viscous triple-deck problem is closed by the relation

$$\frac{\partial P}{\partial x} = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{A''(\zeta)}{x-\zeta} \, d\zeta, \tag{2.5}$$

between the pressure $P(x)$ and the displacement function $A(x)$. Also note that even though the governing equation (2.2) is parabolic, the pressure–displacement interaction law (2.5) makes the whole problem elliptic.

The analytical structure of the underlying problem for the interaction region in a variety of physical situations leads, almost invariably, to the same fundamental equation (2.2) with only a small change necessary in the boundary conditions. Computations in this paper have been performed with the shape function characterizing the form of the trailing edge given by

$$H(x) = \begin{cases} -\beta x & \text{for } x < 0, \\ 0 & \text{for } x > 0. \end{cases} \tag{2.6}$$

The large asymptotic behaviour of the displacement function, as well as the pressure far upstream and downstream of the aerofoil, can be derived as in Sychev *et al.* (1998), and they are

$$A(x) \to \beta x + O(x^{-1/3}), \quad x \to -\infty, \qquad A(x) \to \gamma x^{1/3}, \quad x \to \infty, \tag{2.7}$$

$$P(x) \to -(\beta/\pi) \ln(x), \quad x \to \pm\infty, \tag{2.8}$$

where $\gamma = 0.89$ and β is the scaled wedge angle, which is connected to the physically small angle ϕ through $\beta = Re^{1/4}\lambda^{-1/2}\phi$.

Numerical solutions were also obtained for the triple-deck flow over humps and near a concave corner, for which the shape function $H(x)$ is given by

$$H(x) = \begin{cases} 0 & \text{for } x < 0, \\ \beta x & \text{for } x > 0. \end{cases} \tag{2.9}$$

The hump shape is defined as in Smith & Bodonyi (1985):

$$H(x) = \begin{cases} h(1-x^2)^2 & \text{for } -1 < x < 1, \\ 0 & \text{elsewhere.} \end{cases} \tag{2.10}$$

The boundary conditions (2.3) and the far-downstream asymptotes (2.7), (2.8) for the flow over a corner and hump are also suitably modified, and are given, for example, in Smith & Merkin (1982). In (2.9) and (2.10), β is a scaled corner angle, and h a scaled hump size parameter.

(*c*) *Numerical method of solution*

For the numerical treatment of the trailing-edge flow problem just described, we map the infinite physical region onto a finite computational domain by the transform $x = \tan(Z)$, so that the constraints far upstream and far downstream could be set exactly at $Z = \pm\frac{1}{2}\pi$, where $|x| \to \infty$, and the irregular behaviours are treated adequately in equation (2.7). In order to avoid the growing properties of the thickness and the pressure in (2.7) and (2.8), we introduce the following transformations:

$$\left.\begin{aligned} A &= \frac{\beta}{\pi}[x(\tfrac{1}{2}\pi - Z)] + \frac{2\gamma}{3^{1/2}}(1+x^2)^{1/6}\sin(\tfrac{1}{6}(5\pi - 2Z)) + A_{\mathrm{s}}, \\ P &= -\frac{\beta}{2\pi}\ln(1+x^2) + \frac{\beta}{\pi(1+x^2)} - \frac{2\gamma}{3^{3/2}}(1+x^2)^{-1/3}\cos(\tfrac{1}{3}(\pi + 2Z)) + P_{\mathrm{s}}. \end{aligned}\right\} \tag{2.11}$$

This idea was originally introduced in the triple-deck study of Smith & Merkin (1982). Note that with these transformations, P_{s} and $-\mathrm{d}A_{\mathrm{s}}/\,\mathrm{d}x$ remain conjugate pairs and the inviscid interaction law then becomes

$$P_{\mathrm{s}} = \frac{1}{\pi}\int_{-\pi/2}^{\pi/2} \frac{A_{\mathrm{s}}'(\zeta)}{\tan(Z) - \tan(\zeta)}\,\mathrm{d}\zeta. \tag{2.12}$$

Working with A_{s} instead of A ensures that A_{s} decays asymptotically as $Z \to \pm\frac{1}{2}\pi$, so that the integral equation (2.12) may be handled in a more systematic manner. In this way, the elliptic inviscid law is treated with a method introduced by Veldman (1979) in which the integration (2.12) is carried out at N local points leading to

$$P_{\mathrm{s}}(x_j) = \sum_{i=0}^{N} \beta_{ij} A_{\mathrm{s}i},$$

where

$$\beta_{ij} = \begin{cases} -f_{1j}, & i = 0, \\ f_{i-1j} - f_{ij}, & 1 \leqslant i \leqslant N-1, \\ f_{N-1j}, & i = N, \end{cases}$$

and $f_{ij} = [\tan(x_j) - \tan(x_{i+(1/2)})]^{-1}$. We choose this kind of approximation to the integral owing to the fact that it handles the singularity effectively by using implicit approximation of the interaction condition (2.12), and it remains superior to the other known methods, the so-called inverse or semi-inverse iterative methods, as far as the convergence rate is concerned.

The nonlinear interaction equation (2.2) is first differentiated with respect to $\bar{Y}$ to eliminate the pressure gradient. To complete the system we need one more equation, which may be obtained from equation (2.2) by setting $\bar{Y} = 0$. For the trailing-edge flow, for instance, we obtain:

$$P_x = \begin{cases} \psi'''(0), & x \leqslant 0, \\ \psi'''(0) - \psi'(0)\psi_x'(0), & x > 0. \end{cases} \tag{2.13}$$

Equations (2.2)–(2.13) are discretized using a Chebyshev collocation method (see, for example, Canuto *et al.* 1988) in the $\bar{Y}$-direction, after truncating the semi-infinite physical domain at a value $\bar{Y}_{\max}$ and mapping onto the $[-1, 1]$ Chebyshev computational domain by means of a linear transformation. Standard second-order finite differences are used for the x-derivatives. This yields a nonlinear system, which is then linearized with a suitable initial guess using the Newton linearization technique. Finally, the resulting linear system together with the boundary conditions are combined in a generalized matrix form, which is then solved using LU decomposition.

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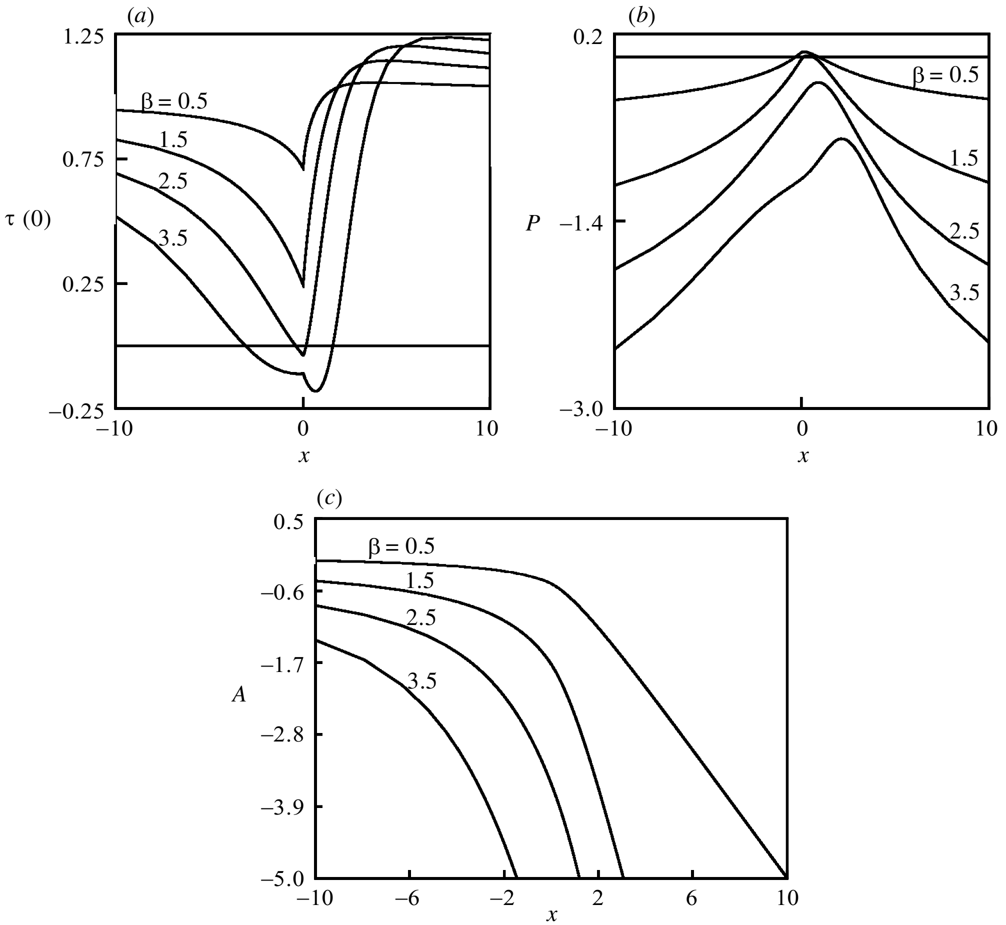

Figure 2. Corner-flow results are shown for a various values of β. (*a*) Skin friction along the surface of the plate, (*b*) pressure distribution, and (*c*) displacement function.

3. Mean-flow results

The numerical results obtained by solving the fundamental triple-deck equations are presented in this section. Typically, 200 streamwise locations in the finite interval $[-\frac{1}{2}\pi, \frac{1}{2}\pi]$ and 64 Chebyshev collocation points were used. The edge of the outer boundary $\bar{Y}_{\max}$ was chosen to be 10 for all the calculations. To check on the effect of the computational grid, these parameters were halved or doubled accordingly for each flow calculation. The code was first verified by computing results for the triple-deck flow over a corner and also the triple-deck flow over a hump.

A numerical solution of the nonlinear interaction problem for both cases has also been obtained by Ruban (1976, 1977). Smith & Merkin (1982) extended the calculations for larger values of β and h. Our results are shown in figures 2 and 3.

In general, good agreement is found with the results of Ruban (1976, 1977), Smith & Merkin (1982) and Smith & Bodonyi (1985). However, Ruban (1976) concluded that in the case of a concave corner, flow separation first appears when $\beta = 2$ (see also Sychev *et al.* 1998). Yet from the calculations of Smith & Merkin (1982) (see

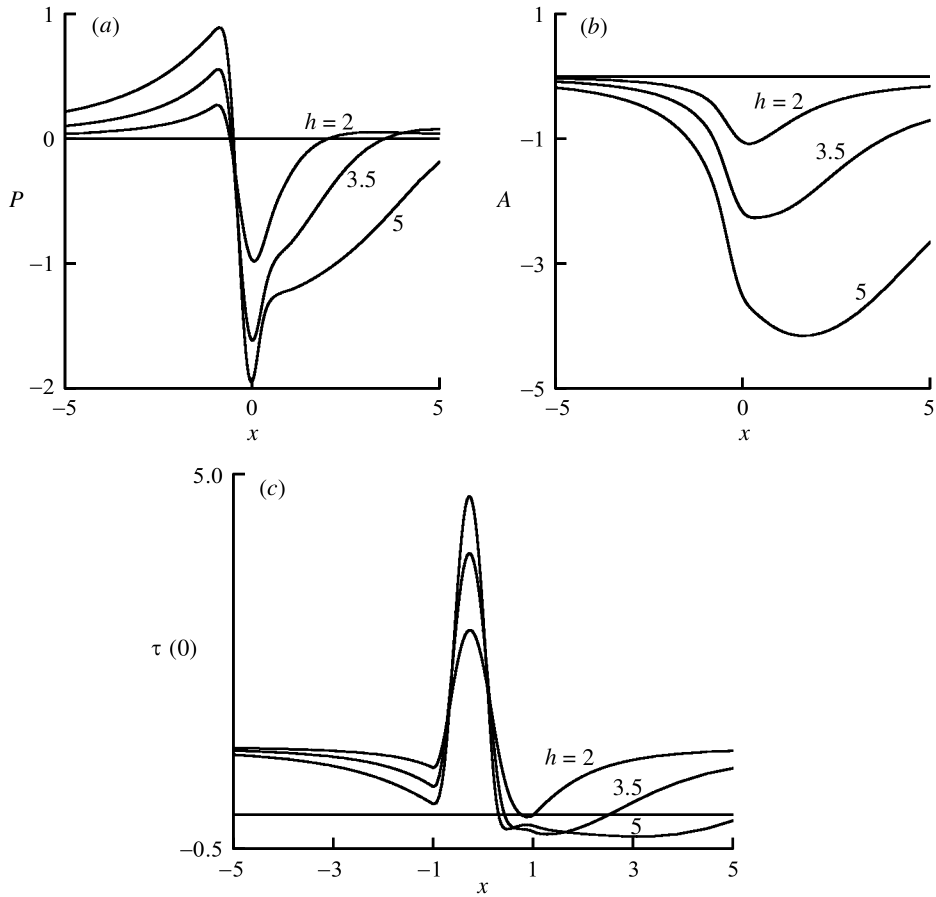

Figure 3. Results for the flow over the hump defined by (2.10) are shown for a various values of h. (*a*) Pressure distribution, (*b*) displacement function, and (*c*) skin friction.

fig. 5 in their paper), separation is found to occur at a larger value of $\beta = 2.51$. From our calculations we find that separation is encountered around $\beta = 2.09$, which is relatively close to the value found by Ruban.

(*a*) *Wedged trailing-edge results*

Calculations were next carried out for the wedged trailing-edge flow. Figure 4 shows the distribution of the skin friction and centreline velocity along the wake axis of symmetry for a range of values of scaled wedge angle β between 0 and 3.8. As β increases, separation occurs and a recirculating zone of fluid particles appears in the vicinity of the trailing edge, the extent of which increases with increasing β. From figure 5*a* the critical parameter leading to a change from attached to separated flow appears, from our calculation, to be 2.56, which is in quite good agreement with the value of 2.6 obtained by Ruban (1977). Smith & Merkin (1982) found that separation first occurs for $\beta = 2.38$ for the same reduced shape (see fig. 7 in their paper). The corresponding dependence of the distribution on the pressure as well as the displacement thickness on x for given values of β is shown in parts (*b*) and (*c*) of figure 5.

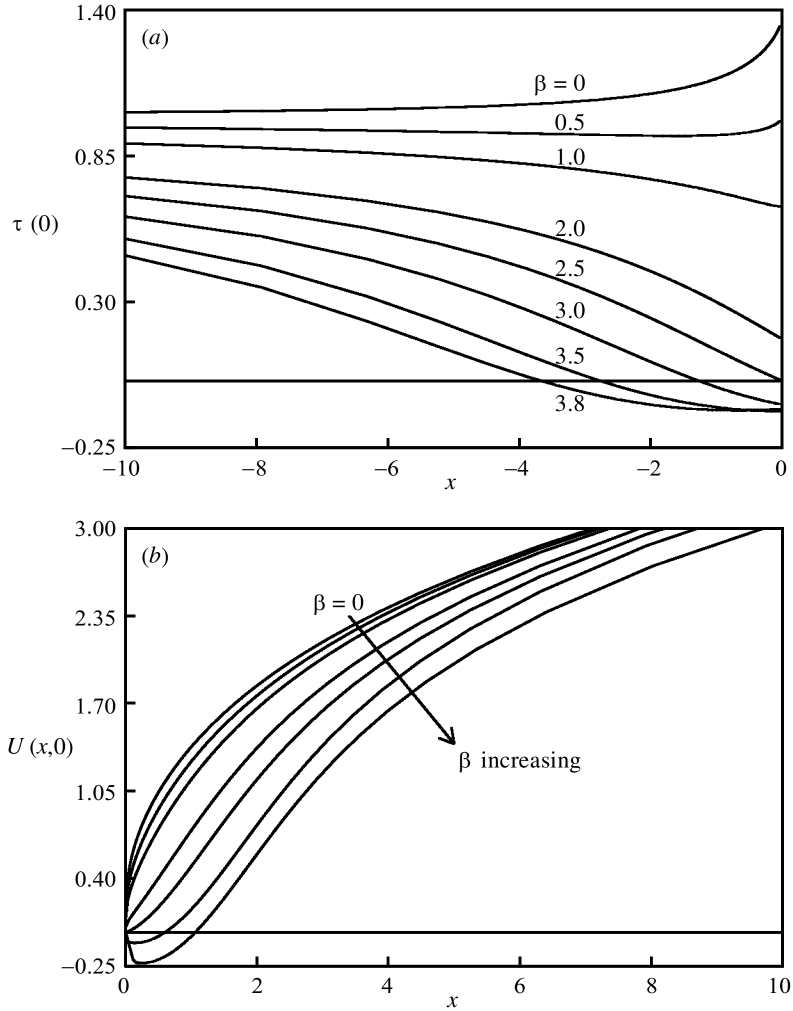

Figure 4. Skin friction on the plate and centreline velocity along the wake are shown for a range of values of wedge angle parameter β.

Overall, these results agree well with the published results of Jobe & Burggraf (1974) and Ruban (1977), but differ from Smith & Merkin (1982), particularly for values of β close to separation. It is noted that our method of solution is different from those of Ruban (1976, 1977) and Smith & Merkin (1982). In our method we used Chebyshev collocation to approximate the vertical derivatives instead of finite differencing used in those latter papers. We believe that the differences between the results of Smith & Merkin (1982) and our results arise from errors introduced in the transformed wake symmetry boundary conditions that Smith & Merkin (1982) employed in their calculations.

An increase in the parameter β leads to the pressure gradient becoming adverse and the separated flow region is pushed upstream ahead of the trailing edge. Although the recirculating region gets widened, the fluid particles here possess a comparably small value of velocity. We were only able to compute the solutions up to $\beta = 3.8$,

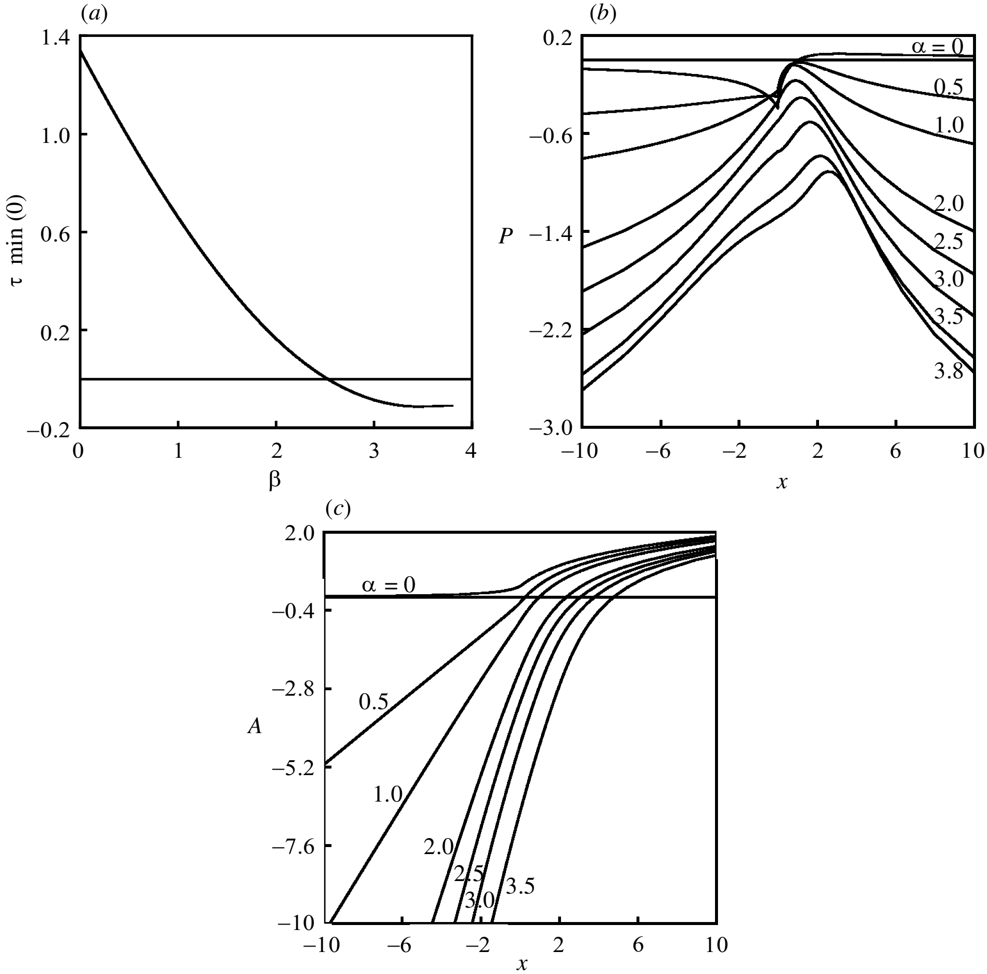

Figure 5. Wedged trailing-edge flow results are shown for a range of values of β. (*a*) Variation of the minimum value of the skin friction with respect to β, (*b*) pressure distribution, (*c*) displacement function.

after which our numerical procedure failed. As seen from figure 5*a*, the minimum skin friction reaches a minimum and then shows a tendency to increase slightly as β approaches 3.8. This sort of phenomenon is also encountered in the work of Korolev (1991, 1992) for the corner flow and he explains this behaviour as hysteresis, implying non-existence of the solutions beyond this critical value. It also implies that the solution of the interaction problem may exist only within a certain range of the angle parameter β, and multiple solutions of the problem beyond this range may be possible. It would be useful to extend these calculation using more direct methods (as in Korolev (1992)) and study possible hysteresis effects of the separated flow.

(*b*) *Composite solutions*

Having solved the triple-deck equations, it is now an easy task to construct asymptotic composite solutions uniformly valid in the triple-deck region. Due to small contributions from the upper-deck expansion, only the main-deck and lower-deck solutions will be considered in the formation of a composite solution. It will, however, be Reynolds number dependent, and here we suppose that the Reynolds number is measured based on the main chord-line length of the aerofoil. Let U_{m} denote the main-deck solution given by (2.1), U_{l} the lower-deck solution to be obtained from (2.2)–(2.5), and U_{mtc} the matching between them. Upon matching the viscous wall layer solution with (2.1) we find that

$$U_{\mathrm{mtc}}(Y) = \lambda(Y + Re^{-1/8}(A(X) + H(X))) \quad \text{as } Y \to \infty. \tag{3.1}$$

Taking into account that $\bar{Y} = Re^{1/8}Y$, where $\bar{Y}$ is the lower-deck boundary-layer scale, we obtain the solution for the composite expansion as

$$U(Y) = U_{\mathrm{B}}(Y) + Re^{-1/8}(A(X) + H(X))U'_{\mathrm{B}}(Y) + Re^{-1/8}U_{\mathrm{l}}(Re^{1/8}Y) \\ - \lambda(Y + Re^{-1/8}(A(X) + H(X))). \tag{3.2}$$

Notice that this solution satisfies the matching requirement used in the boundary condition (2.4), as well as $U(Y = 0) = 0$ and $U(Y \to \infty) \to 1$.

Sample composite mean velocity profiles obtained from equation (3.2) for $Re = 500$ are shown in figure 6 for the wedged trailing-edge flow. Each graph here corresponds to $\alpha_\star = 0$, 0.5, 1.5 and 2, respectively, where $\alpha_\star = \sqrt{\lambda}\beta$. It is seen that these profiles correspond to the Gaussian type of wake profiles far downstream. One can also see from the third portion of figure 6 that the flow separation has already started at $\alpha_\star = 1.5$. Therefore, the triple-deck velocity profiles obtained here have the advantage that the linear stability properties may be investigated even for the regions of flow reversal. This is discussed in the next section.

4. Absolute instability results

Smith & Bodonyi (1985) have shown that the triple-deck profiles in the boundary-layer flow past an obstacle on a surface are inviscidly unstable. Because the triple-deck solutions in the separation zone for this kind of flow give rise to inflectional profiles, they were able to compute the inviscid instability properties, on the grounds that the lower-deck modes have spatial or temporal growth rates much higher than those associated with the main- or upper-decks, due to the inflectional velocity profiles having the shortest scale in physical variables x and y (both $O(Re^{-5/8})$. This leads to the inviscid Rayleigh equation (see Smith & Bodonyi 1985). The nonlinear results presented in Duck (1985, 1988) indicated that rapid growth of the spectral solution occurs for large wavenumbers, suggesting the development of a short-scale Rayleigh-type instability. Taking this into account, we first attempted to investigate whether the wedged trailing-edge flow also has inflectional lower-deck velocity profiles. However, these profiles do not appear to have any inflection points. This may be solely because acceleration of the fluid particles is possible in the trailing-edge flow, whereas acceleration of the fluid particles over the obstacle, followed by deceleration, seemingly raises the occurrence of inflectional points in the boundary-layer flow past wall distortions.

The presence of inflectional instability suggests the possibility that absolute instability may also exist. The lower-deck velocity profiles for the flow over the hump

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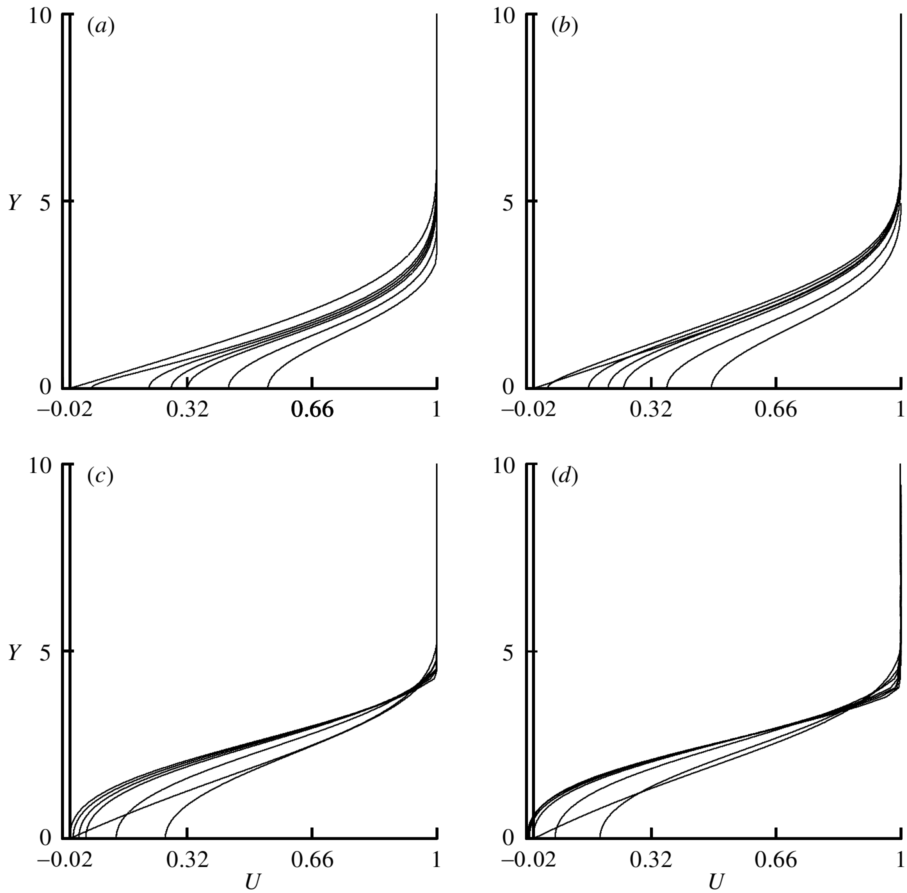

Figure 6. Composite velocity profiles of the wedged trailing-edge flow are shown for $Re = 500$. Curves correspond to the x locations (from left to right) -32.0, 0.01, 0.5, 1.0, 1.5, 3.1 and 5.2, respectively.

shape (2.10) were investigated using our inviscid Rayleigh equation solver. In figure 7*a* the presence of a saddle-point-type phenomenon is demonstrated. Figure 7*b* shows that the saddle point in figure 7*a* is a pinch point with $\omega_i > 0$, suggesting absolute instability. Here $\omega = \omega_r + i\omega_i$, $\alpha = \alpha_r + i\alpha_i$ are the scaled complex frequency and wavenumber of the disturbances, respectively. In figure 7*c*, *d*, branch points are given in the α, ω-planes for different hump sizes.

According to figure 7*c*, the instability starts almost at the peak point for each hump size and extends far downstream of the hump. Increasing the disturbance height also causes an increase in the absolute growth rate of the disturbances, as might be expected physically. We suggest, therefore, that the instability observed by Duck (1985, 1988) leading to a finite-time breakdown of the lower-deck boundary-layer solution may be related to the absolute instability found here.

The flow in the vicinity of the trailing edge changes most rapidly, and, therefore, it might be expected that the viscous triple-deck region close to the trailing edge would exhibit a strong influence on the stability properties. For this reason the composite basic velocity profiles obtained from the triple-deck mean flow are considered next.

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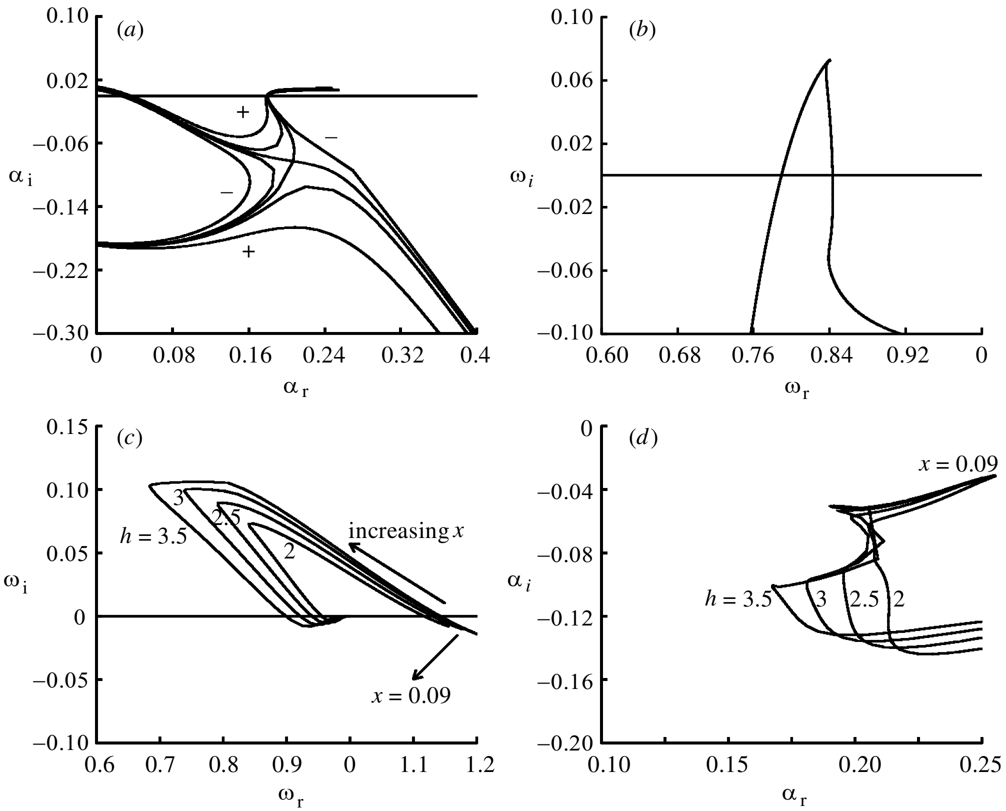

Figure 7. A branch point satisfying the pinching criterion is shown ((*a*) and (*b*)) in the α- and ω-planes, for hump height $h = 2$ and x station $x = 0.94$. Pinching occurs at $\alpha = (0.21, -0.08)$ and $\omega = (0.8, 0.07)$. $(+)$ and $(-)$ show, respectively, increasing and decreasing ω_{i}. Branch points are demonstrated ((*c*) and (*d*)) along the triple-deck region in the α- and ω-planes for hump sizes $h = 2$, 2.5, 3 and 3.5, respectively.

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As displayed in figure 6, the composite velocity profiles apparently have inflectional points somewhere in the middle of the boundary layer. This motivated us to explore the inviscid instability characteristics of such profiles. These profiles were fed into the inviscid Rayleigh solver and afterwards branch points were located. An initial search for branch points on the wedge ahead of the trailing edge did not yield any absolutely unstable branch points, suggesting that the flow on the wedge is only convectively unstable. Figure 8 shows branch points that satisfy the Briggs–Bers criterion for the flow in the wake. (In the wake region there is a change in the boundary conditions and results are shown for disturbances that satisfy the condition of zero pressure on the centreline.) The solid curves are for $Re = 500$, the dashed curves for $Re = 1000$, and the dotted ones for $Re = 10000$, plotted for several values of the parameter $\alpha_{\star}$. It is seen that for $\alpha_{\star}$ larger than 0.5 the wake is absolutely unstable.

We also observe from figure 8 that, for the zero pressure gradient case, there is only convective instability ($\omega_{\mathrm{i}} < 0$), in contrast to the suggestions made by Woodley & Peake (1997), who speculated that the double Blasius profile might be absolutely unstable. The effect of increasing the wedge angle is to have a higher absolute growth

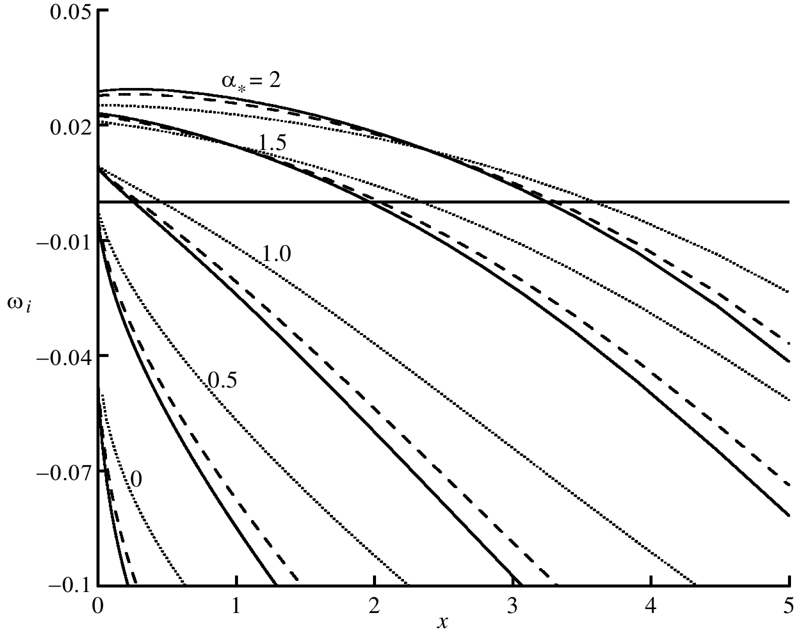

Figure 8. Absolute instability range for the flow over a wedged trailing edge is shown along the triple-deck region for several values of parameter $\alpha_\star$. Solid lines denote $Re = 500$, dashed lines denote $Re = 1000$, and dotted lines denote $Re = 10\,000$.

rate and to make the region of absolute instability much wider. This also implies that separation ensures the existence of absolute instability by enhancing the instability region, at least for the underlying picture of the boundary-layer flow considered here. Increasing the wedge angle parameter β is akin to increasing the aerofoil thickness, which is likely to promote stronger separation.

5. Summary and conclusions

In this paper we have considered both the solutions of the triple-deck equations and the stability of the solutions for the flows over a wall irregularity, a corner, and a wedged trailing edge. The calculations show that separation from the surface of the body begins, respectively, at $\beta = 2.09$ and 2.56 for the corner and wedged trailing edge. These values have been found to compare well with the ones obtained by Ruban (1976, 1977). The maximum wedge angle for which we were able to obtain results is $\beta = 3.8$. The work of Korolev (1991, 1992) would suggest that separation hysteresis may arise after this critical value. Our inviscid stability results have shown that absolute instability exists in the triple-deck flow over the hump and in the wake region behind a wedged trailing edge.

Composite basic velocity profiles have been constructed for a wedged trailing edge, including some wedge angles leading to flow separation. The double Blasius profile that was suggested to be absolutely unstable by Woodley & Peake (1997) has been found to be convectively unstable in the triple-deck region. Other composite velocity profiles have been found to exhibit similar absolute instability characteristics, as found from the profiles from the classical boundary-layer equations. We have found that increasing the wedge angle creates a zone of separated flow which in turn ensures a much larger extent of absolutely unstable flow only *behind* the trailing edge.

The current method can also be used to solve the triple-deck equations for the supersonic corner and wedged trailing-edge flows. For the supersonic flows, the problem reduces to an even simpler partial differential form, since the global interaction law is replaced by Ackeret's formula. Work on computing solutions for the supersonic flow and for non-aligned aerofoils is currently in progress (see also Türkyılmazoğlu *et al.* 1999).

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