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On the absolute instability of the triple-deck On the absolute instability of the triple-deck
flow over humps and near wedged trailing edges

BY J. S. B. GAJJAR AND M. TÜRKYILMAZOĞLU
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The triple-deck equations for the flow over a hump, a corner and a wedged trailing The triple-deck equations for the flow over a hump, a corner and a wedged trailing edge are solved numerically using a novel method based on spectral collocation. It is found that for the flow over a corner, separation be The triple-deck equations for the flow over a hump, a corner and a wedged trailing edge are solved numerically using a novel method based on spectral collocation. It is found that for the flow over a corner, separation be is found that for the flow over a corner, separation begins at a scaled angle β of 2.09, and for the wedged trailing edge for a wedge angle of 2.56. Here β is defined in is found that for the flow over a corner, separation begins at a scaled angle β of 2.09, and for the wedged trailing edge for a wedge angle of 2.56. Here β is defined in terms of the small physical angle ϕ by β 2.09, and for the wedged trailing edge for a wedge angle of 2.56. Here β is defined in
terms of the small physical angle ϕ by $\beta = Re^{1/4}\lambda^{-1/2}\phi$, $\lambda = 0.3320$, and Re is the
Reynolds number. The absolute instability terms of the small physical angle ϕ by $\beta = Re^{1/4}\lambda^{-1/2}\phi$, $\lambda = 0.3320$, and Re is the Reynolds number. The absolute instability of the nonlinear mean flows computed is investigated. It is found that the flow over a hu Reynolds number. The absolute instability of the nonlinear mean flows computed is investigated. It is found that the flow over a hump is inviscidly absolutely unstable with the maximum absolute unstable growth rate occurri investigated. It is found that the flow over a hump is inviscidly absolutely unstable
with the maximum absolute unstable growth rate occurring near the maximum height
of the hump, and increasing with hump size. The wake re with the maximum absolute unstable growth rate occurring near the maximum height
of the hump, and increasing with hump size. The wake region behind the wedged
trailing edge is also found to be absolutely unstable beyond a of the hump, and increasing with hump size. The wake region behind the wedged
trailing edge is also found to be absolutely unstable beyond a critical wedge angle,
and the extent of the region of absolute instability increa trailing edge is also found to be absolutely unstable beyond a critical wedge angle, and the extent of the region of absolute instability increases with increasing wedge angle and separation.

Keywords: boundary layer; separation; stability; triple deck

1. Introduction

1. Introduction
It has been known since Goldstein's (1930) paper that classical boundary-layer the-
ory breaks down pear the trailing edge of a finite flat plate, owing to the presence of It has been known since Goldstein's (1930) paper that classical boundary-layer the-
ory breaks down near the trailing edge of a finite flat plate, owing to the presence of
a large induced pressure gradient there. In order It has been known since Goldstein's (1930) paper that classical boundary-layer the-
ory breaks down near the trailing edge of a finite flat plate, owing to the presence of
a large induced pressure gradient there. In order behaviour and provide continuation of the Blasius solution into the wake, new scal-
behaviour and provide continuation of the Blasius solution into the wake, new scalings need to be introduced. Stewartson (1969, 1970), Neiland (1969) and Messiter behaviour and provide continuation of the Blasius solution into the wake, new scalings need to be introduced. Stewartson (1969, 1970), Neiland (1969) and Messiter (1970) have derived a rational asymptotic expansion of the ings need to be introduced. Stewartson (1969, 1970), Neiland (1969) and Messiter (1970) have derived a rational asymptotic expansion of the flow variables near the trailing edge, and the resulting disturbance structure tha (1970) have derived a rational asymptotic expansion of the flow variables near the trailing edge, and the resulting disturbance structure that they discovered is known as the triple-deck structure. The same structure aris trailing edge, and the resulting disturbance structure that they discovered is known
as the triple-deck structure. The same structure arises in many other related contexts
including near the separation point in an adverse as the triple-deck structure. The same structure arises in many other related contexts
including near the separation point in an adverse pressure gradient boundary layer,
in shock-wave boundary-layer interactions, and in t including near the separation point in an adverse pressure gradient boundary layer,
in shock-wave boundary-layer interactions, and in the stability of boundary layers.
A comprehensive account of the origins and applicatio in shock-wave boundary-layer interactions, and in the stability of boundary la
A comprehensive account of the origins and applications of triple-deck theory
be found in Stewartson (1974, 1981), Smith (1982) and Sychev *et* A comprehensive account of the origins and applications of triple-deck theory may
be found in Stewartson (1974, 1981), Smith (1982) and Sychev *et al.* (1998).
Our concern in this paper is primarily with the flow near the

be found in Stewartson (1974, 1981), Smith (1982) and Sychev *et al.* (1998).
Our concern in this paper is primarily with the flow near the trailing edge of
a wedge-shaped aerofoil. Various numerical methods have been dev Our concern in this paper is primarily with the flow near the trailing edge of
a wedge-shaped aerofoil. Various numerical methods have been developed to solve
the equations governing triple-deck viscous-inviscid interactio a wedge-shaped aerofoil. Various numerical methods have been developed to solve
the equations governing triple-deck viscous-inviscid interaction problems. For the
trailing-edge flow, the presence of an abrupt discontinuity the equations governing triple-deck viscous-inviscid interaction problems. For the trailing-edge flow, the presence of an abrupt discontinuity in the boundary conditions at the trailing edge, and also the interaction law r trailing-edge flow, the presence of an abrupt discontinuity in the boundary condi-
tions at the trailing edge, and also the interaction law relating the pressure and the
displacement thickness of the boundary layer for su tions at the trailing edge, and also the interaction law relating the pressure and the tions using a numerical marching procedure to calculate the symmetric flow near the

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trailing edge of a flat plate at zero incidence and the asymmetric supersonic flow trailing edge of a flat plate at zero incidence and the asymmetric supersonic flow
over a flat plate at an angle of attack. He also computed asymptotic solutions for
near-trailing-edge and downstream flow. The first numeri trailing edge of a flat plate at zero incidence and the asymmetric supersonic flow
over a flat plate at an angle of attack. He also computed asymptotic solutions for
near-trailing-edge and downstream flow. The first numeri over a flat plate at an angle of attack. He also computed asymptotic solutions for
near-trailing-edge and downstream flow. The first numerical solution of the interac-
tion problem for subsonic flow near the trailing edge near-trailing-edge and downstream flow. The first numerical solution of the interaction problem for subsonic flow near the trailing edge of a flat plate was given by Jobe & Burggraf (1974). They proposed a combined inverse tion problem for subsonic flow near the trailing edge of a flat plate was given by
Jobe & Burggraf (1974). They proposed a combined inverse method in which, for
a given displacement distribution, the pressure was computed Jobe & Burggraf (1974). They proposed a combined inverse method in which, for
a given displacement distribution, the pressure was computed and the displacement
updated using this new pressure. Their calculations showed th a given displacement distribution, the pressure was computed and the displacement
updated using this new pressure. Their calculations showed that an acceleration of
the fluid ahead of the trailing edge of the plate is alw updated using this new pressure. Their calculations showed that an acceleration of
the fluid ahead of the trailing edge of the plate is always present due to the singu-
lar behaviour of an adverse pressure gradient. Chow the fluid ahead of the trailing edge of the plate is always present due to the singular behaviour of an adverse pressure gradient. Chow & Melnik (1976) applied this method for calculating the asymmetric flow over the trail lar behaviour of an adverse pressure gradient. Chow & Melnik (1976) applied this
method for calculating the asymmetric flow over the trailing edge of a flat plate at an
angle of attack. Smith (1974) developed a two-region method for calculating the asymmetric flow over the trailing edge of a flat plate at an
angle of attack. Smith (1974) developed a two-region matching procedure that very
closely follows the mathematical development of the angle of attack. Smith (1974) developed a two-region matching procedure that very closely follows the mathematical development of the double-layered solution near the singularity. Making use of this method, he solved the closely follows the mathematical development of the double-layered solution near the singularity. Making use of this method, he solved the problem of slot injection into the fluid from a flat plate. Daniels (1974a, b) exp flows. the fluid from a flat plate. Daniels $(1974a, b)$ exploited this technique for trailing-edge flows.
The numerical solution of the nonlinear interaction problem was also obtained

by Ruban (1976, 1977). His numerical method was based on solving the boundary-The numerical solution of the nonlinear interaction problem was also obtained
by Ruban (1976, 1977). His numerical method was based on solving the boundary-
layer equations for a given displacement thickness and combined by Ruban (1976, 1977). His numerical method was based on solving the boundary-
layer equations for a given displacement thickness and combined with the method
of Jobe & Burggraf (1974). Using this method, Ruban (1976, 1977 layer equations for a given displacement thickness and combined with the method
of Jobe & Burggraf (1974). Using this method, Ruban (1976, 1977) calculated sep-
aration occurring around a surface irregularity and near the of Jobe & Burggraf (1974). Using this method, Ruban (1976, 1977) calculated separation occurring around a surface irregularity and near the trailing edge of a thin symmetric aerofoil. Later, both Ruban & Sychev (1979) and aration occurring around a surface irregularity and near the trailing edge of a thin
symmetric aerofoil. Later, both Ruban & Sychev (1979) and Smith & Merkin (1982)
investigated the triple-deck flow around a wedge-shaped t symmetric aerofoil. Later, both Ruban & Sychev (1979) and Smith & Merkin (1982)
investigated the triple-deck flow around a wedge-shaped trailing edge. The latter also
extended their studies to the viscous-inviscid interact investigated the triple-deck flow around a wedge-shaped trailing edge. The latter also
extended their studies to the viscous-inviscid interaction due to a small hump and
convex/concave corners. They transformed the infinit extended their studies to the viscous-inviscid interaction due to a small hump and
convex/concave corners. They transformed the infinite physical domain completely
into a finite range of streamwise integration, and simple convex/concave corners. They transformed the infinite physical domain completely
into a finite range of streamwise integration, and simple transformations were made
for the pressure and displacement function to prevent the into a finite range of streamwise integration, and simple transformations were made
for the pressure and displacement function to prevent the original unbounded growth
of these two functions. They investigated separation t for the pressure and displacement function to prevent the original unbounded growth of these two functions. They investigated separation taking place in an incompressible flow near a corner of a body, near the wedge-shaped of these two functions. They investigated separation taking place in an incompressangles β , 2.51, -5.21 and 2.38, for the onset of the separation at a concave corner, a aerofoil, and some other external flow structures. Their results predicted the scaled angles β , 2.51, -5.21 and 2.38, for the onset of the separation at a concave corner, a convex corner, and a wedged trailing edge, re angles β , 2.51, -5.21 and 2.38, for the onset of the separation at a concave corner, a
convex corner, and a wedged trailing edge, respectively. The first and last values are
somewhat different from the corresponding va convex corner, and a wedged trailing edge, respectively. The first somewhat different from the corresponding values of 2.0 and 2.6 by Ruban (1976, 1977) and also given in Sychev *et al.* (1998). The calculations performed mewhat different from the corresponding values of 2.0 and 2.6 that were computed
Ruban (1976, 1977) and also given in Sychev *et al.* (1998).
The calculations performed by Ruban (1976, 1977), Ruban & Sychev (1979) and
ait

by Ruban (1976, 1977) and also given in Sychev *et al.* (1998).
The calculations performed by Ruban (1976, 1977), Ruban & Sychev (1979) and
Smith & Merkin (1982) were based on the use of iterative schemes which became
div The calculations performed by Ruban (1976, 1977), Ruban & Sychev (1979) and
Smith & Merkin (1982) were based on the use of iterative schemes which became
divergent when the region of recirculating flow was sufficiently la Smith & Merkin (1982) were based on the use of iterative schemes which became
divergent when the region of recirculating flow was sufficiently large. To study the
behaviour of the solution at larger values of β for the divergent when the region of recirculating flow was sufficiently large. To study the behaviour of the solution at larger values of β for the corner flow, Korolev (1991, 1992) employed a direct method and calculated a r behaviour of the solution at larger values of β for the corner flow, Korolev (1991, 1992) employed a direct method and calculated a recirculating zone up to $\beta = 7$. A further increase in β created a singularity in 1992) employed a direct method and calculated a recirculating zone up to β further increase in β created a singularity in the skin friction immediately a the attachment point, and thus made the use of interaction th The state interest is the stability in the skin friction immediately ahead of eattachment point, and thus made the use of interaction theory invalid.
Also of great interest is the stability of the locally distorted steady

the attachment point, and thus made the use of interaction theory invalid.
Also of great interest is the stability of the locally distorted steady or unsteady separated flow motions. In addition to the Tollmien–Schlichting Also of great interest is the stability of the locally distorted steady or unsteady separated flow motions. In addition to the Tollmien–Schlichting modes of instability, the triple-deck solutions admit inflectional velocit arated flow motions. In addition to the Tollmien–Schlichting modes of instability, the
triple-deck solutions admit inflectional velocity profiles which are susceptible to an
inviscid Rayleigh-type instability. Rayleigh ins triple-deck solutions admit inflectional velocity profiles which are susceptible to an inviscid Rayleigh-type instability. Rayleigh instability has typical length-scale much shorter than that of Tollmien–Schlichting instab nomenon due to the fairly sudden production of faster spatial and temporal growth shorter than that of Tollmien–Schlichting instability and represents a bursting phenomenon due to the fairly sudden production of faster spatial and temporal growth effects. Smith $\&$ Bodonyi (1985) have shown that the t nomenon due to the fairly sudden production of faster spatial and temporal growth effects. Smith $\&$ Bodonyi (1985) have shown that the triple-deck solutions over a mounted-surface obstacle are subjected to a short-scale mounted-surface obstacle are subjected to a short-scale Rayleigh-type instability,
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provided that the obstacle size is sufficiently large to generate nonlinear solutions.
Using temporal stability theory, they have computed the temporal growth rates at different locations corresponding to flow-reversal reg **MATHEMATICAL,
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SCIENCES provided that the obstacle size is sufficiently large to generate nonlinear solutions. provided that the obstacle size is sufficiently large to generate nonlinear solutions.
Using temporal stability theory, they have computed the temporal growth rates at
different locations corresponding to flow-reversal reg

Using temporal stability theory, they have computed the temporal growth rates at different locations corresponding to flow-reversal regions. Duck (1985, 1988) has also encountered an inflectional instability in unsteady in different locations corresponding to flow-reversal regions. Duck (1985, 1988) has also
encountered an inflectional instability in unsteady incompressible boundary-layer
flow computations, which manifested itself in such a encountered an inflectional instability in unsteady incompressible boundary-layer
flow computations, which manifested itself in such a way that large wavenumbers
were excited by surface distortions, leading to the breakdow flow computations, which if
were excited by surface dis-
finite time is approached.
The present work is cone were excited by surface distortions, leading to the breakdown of the solution as a finite time is approached.
The present work is concerned with the triple-deck flows over humps, a concave

corner, and wedged trailing-edge configurations. There are several main objectives. The present work is concerned with the triple-deck flows over humps, a concave
corner, and wedged trailing-edge configurations. There are several main objectives.
The first is to develop a viscous-inviscid interaction code corner, and wedged trailing-edge configurations. There are several main objectives.
The first is to develop a viscous-inviscid interaction code using spectral methods,
which have the distinct advantage that greater accurac The first is to develop a viscous–inviscid interaction code using spectral methods, which have the distinct advantage that greater accuracy is easily obtained with only a modest increase in the number of points used. Anot which have the distinct advantage that greater accuracy is easily obtained with only
a modest increase in the number of points used. Another aim is to compute solutions
for the wedged trailing-edge flows with the scaled a a modest increase in the number of points used. Another aim is to compute solutions
for the wedged trailing-edge flows with the scaled angle parameter β larger than
those previously computed. Another aspect of the curr for the wedged trailing-edge flows with the scaled angle parameter β larger than
those previously computed. Another aspect of the current work is to resolve the
controversy in the difference, as far as the critical ang those previously computed. Another aspect of the current work is to resolve the controversy in the difference, as far as the critical angle at the onset of separation is concerned, between the two sets of results given by controversy in the difference, as far as the critical angle at the onset of separation is concerned, between the two sets of results given by Ruban (1976, 1977) and Smith $\&$ Merkin (1982). Finally, the work of Smith $\&$ concerned, between the two sets of results given by Ruban (1976, 1977) and Smith $\&$ Merkin (1982). Finally, the work of Smith $\&$ Bodonyi (1985) suggests the occurrence of Rayleigh instability for some classes of nonli Merkin (1982). Finally, the work of Smith & Bodonyi (1985) suggests the occurrence
of Rayleigh instability for some classes of nonlinear triple-deck mean flows. It is well
known that many profiles involving backflow and s of Rayleigh instability for some classes of nonlinear triple-deck mean flows. It is well known that many profiles involving backflow and separation are prone to absolute instability (see Huerre $\&$ Monkewitz 1990; Gaster known that many profiles involving backflow and separation are prone to absolute instability (see Huerre $\&$ Monkewitz 1990; Gaster 1984). Another objective is, thus, to examine the inviscid stability of some of these me instability (see Huerre & Monkewitz 1990; Gaster 1984). Another objective is, thus, examine the inviscid stability of some of these mean flows and to investigate
nether these flows are absolutely unstable or not.
In $\S 2$, the triple-deck equations governing the flow in the vicinity of the trailing
ge ar

whether these flows are absolutely unstable or not.
In $\S 2$, the triple-deck equations governing the flow in the vicinity of the trailing
edge are given and brief details of the numerical method are outlined. Our mean-fl In § 2, the triple-deck equations governing the flow in the vicinity of the trailing edge are given and brief details of the numerical method are outlined. Our mean-flow calculation results are presented in § 3. Some stab edge are given and brief details of the numerical
calculation results are presented in $\S 3$. Some stasum
mary and conclusions are given in $\S 5$.

ummary and conclusions are given in 35 .
2. Problem formulation and solution of the triple-deck equations

2. Problem formulation and solution of the triple-deck equations
Triple-deck theory divides the main region of the flow into three subregions, namely
the lower main and upper decks. The governing equations in each of these Triple-deck theory divides the main region of the flow into three subregions, namely
the lower, main and upper decks. The governing equations in each of these regions
are given in Smith (1982) and Sychev *et al.* (1998) I Triple-deck theory divides the main region of the flow into three subregions, namely
the lower, main and upper decks. The governing equations in each of these regions
are given in Smith (1982) and Sychev *et al.* (1998). I the lower, main and upper decks. The governing equations in each of these regions are given in Smith (1982) and Sychev *et al.* (1998). In order to construct uniformly valid composite solutions, which are used for the sta are given in Smith (1982) and Sychev *et al.* (1998). In order to construct uniformly valid composite solutions, which are used for the stability computations, we will be interested in the expansions in the main and lower valid composite solutions, which are used for the stability computations, we will be interested in the expansions in the main and lower decks only. Note that the contribution to the composite solution from the upper deck interested in the expansions in the main and lower decks only. Note that the contribution to the composite solution from the upper deck will be disregarded, because the effects there are small, $O(Re^{-1/4})$, where Re denot bution to the composite solution from the
the effects there are small, $O(Re^{-1/4})$, who
on the chord-wise extent of the aerofoil.
Consider now a thin symmetric aerofoil e effects there are small, $O(Re^{-1/4})$, where Re denotes the Reynolds number based
the chord-wise extent of the aerofoil, with a sharp wedge-shaped trailing edge,
Consider now a thin symmetric aerofoil, with a sharp wedge

on the chord-wise extent of the aerofoil.
Consider now a thin symmetric aerofc
placed in an incompressible fluid flow.
We assume that the undisturbed freest Consider now a thin symmetric aerofoil, with a sharp wedge-shaped trailing edge,
aced in an incompressible fluid flow.
We assume that the undisturbed freestream uniform flow is parallel to the aerofoil's
ord-line. Later we

placed in an incompressible fluid flow.
We assume that the undisturbed freestream uniform flow is parallel to the aerofoil's
chord-line. Later we select an orthogonal curvilinear coordinate system aligned with
the body sur We assume that the undisturbed freestream uniform flow is parallel to the aerofoil's chord-line. Later we select an orthogonal curvilinear coordinate system aligned with the body surface such that the wake centreline is i chord-line. Later we select an orthogonal curvilinear coordinate system aligned with
the body surface such that the wake centreline is in the streamwise direction. We
confine ourselves to the consideration of only the upp the body surface such the
confine ourselves to the c
account the symmetry.
The two regions—nam mfine ourselves to the consideration of only the upper plane, i.e. $y \ge 0$, taking into
count the symmetry.
The two regions—namely main and lower decks—are depicted in figure 1 for a
edged trailing edge having a typical w

account the symmetry.
The two regions—namely main and lower decks—are depicted in figure 1 for a
wedged trailing edge having a typical wedge angle ϕ . The problem of finding the
asymptotic solution to the Navier-Stokes The two regions—namely main and lower decks—are depicted in figure 1 for a wedged trailing edge having a typical wedge angle ϕ . The problem of finding the asymptotic solution to the Navier-Stokes equations for the spec wedged trailing edge having a typical wedge angle ϕ . The problem of finding the asymptotic solution to the Navier-Stokes equations for the specific flow is tackled using the well-known triple-deck analysis under the li using the well-known triple-deck analysis under the limit $Re \rightarrow \infty$. The full analysis *Phil. Trans. R. Soc. Lond.* A (2000)

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Figure 1. Triple-deck region and scalings are shown in the vicinity of the trailing edge of a wedged Figure 1. Triple-deck region and scalings are shown in the vicinity of the trailing edge of a wedged
shape. ϕ is the wedge angle and is related to scaled wedge angle parameter β by $\beta = \lambda^{-1/2} Re^{1/4} \phi$.
leading to t

be found in Stewartson (1969, 1974), Smith & Merkin (1982) and Sychev *et al.* (1998).

(*a*) *Main-deck solution*

In this region the boundary-layer coordinate Y is linked to the physical coordinate In this region the boundary-layer coordinate Y is linked to the physical coordinate y by the relation $y = Re^{-1/2}Y$. Since we assume that the aerofoil is very thin, the velocity at the outer edge of the boundary layer is no In this region the boundary-layer coordinate Y is linked to the physical coordinate y by the relation $y = Re^{-1/2}Y$. Since we assume that the aerofoil is very thin, the velocity at the outer edge of the boundary layer is no y by the relation $y = Re^{-1/2}Y$. Since we assume that the aerofoil is very thin, the velocity at the outer edge of the boundary layer is not very different from the velocity of the oncoming flow. Therefore, the leading term velocity at the outer edge of the boundary layer is not very different from the velocity
of the oncoming flow. Therefore, the leading term of the velocity expansion in this
regime coincides with the Blasius solution, deno of the oncoming flow. Therefore, the leading term of the velocity expansion in this regime coincides with the Blasius solution, denoted by $U_{\rm B}$ below. Without going into much detail, we write the leading- and second-o regime coincides with the Blasius solution, denoted into much detail, we write the leading- and second-on
streamwise velocity distribution only in the form

$$
U = U_{\mathcal{B}}(Y) + Re^{-1/8}(A(X) + H(X))U'_{\mathcal{B}}(Y).
$$
\n(2.1)

 $U = U_{\rm B}(Y) + Re^{-1/8}(A(X) + H(X))U'_{\rm B}(Y)$. (2.1)
Here, in the second term, $-A(X)$ denotes the local displacement effect of the bound-
ary layer in the viscous-inviscid interaction and $H(X)$ represents the influence of Here, in the second term, $-A(X)$ denotes the local displacement effect of the bound-
ary layer in the viscous-inviscid interaction, and $H(X)$ represents the influence of
the profile thickness on the flow in the boundary la Here, in the second term, $-A(X)$ denotes the local displa
ary layer in the viscous-inviscid interaction, and $H(X)$
the profile thickness on the flow in the boundary layer. the profile thickness on the flow in the boundary layer.
(*b*) *Lower-deck solution*

(b) Lower-deck solution
Here, $y = Re^{-5/8} \bar{Y}$ and, applying the Prandtl transposition theorem, the triple-
ck equations in terms of the stream function ψ governing the lower-deck reduce Here, $y = Re^{-5/8} \bar{Y}$ and, applying the Prandtl transposition theorem, the triple-
deck equations in terms of the stream function ψ governing the lower-deck reduce to

$$
\frac{\partial \psi}{\partial \bar{Y}} \frac{\partial^2 \psi}{\partial x \partial \bar{Y}} - \frac{\partial \psi}{\partial x} \frac{\partial^2 \psi}{\partial \bar{Y}^2} = -\frac{\partial P}{\partial x} + \frac{\partial^3 \psi}{\partial \bar{Y}^3}.
$$
\nThe boundary conditions for the trailing-edge flow are

\n
$$
(2.2)
$$

e boundary conditions for the trailing-edge flow are
\n
$$
\psi = \frac{\partial \psi}{\partial \bar{Y}} = 0 \quad \text{at } \bar{Y} = 0, \quad x < 0, \qquad \psi = \frac{\partial^2 \psi}{\partial \bar{Y}^2} = 0 \quad \text{at } \bar{Y} = 0, \quad x > 0, \qquad (2.3)
$$
\n
$$
\psi = \frac{1}{2}(\bar{Y} + A(x) + H(x))^2 \quad \text{as } \bar{Y} \to \infty, \qquad \psi \to \frac{1}{2}\bar{Y}^2 \quad \text{as } x \to -\infty. \tag{2.4}
$$

$$
\psi = \frac{1}{2}(\bar{Y} + A(x) + H(x))^2 \quad \text{as } \bar{Y} \to \infty, \qquad \psi \to \frac{1}{2}\bar{Y}^2 \quad \text{as } x \to -\infty. \tag{2.4}
$$

In the above, $x = \lambda^{-5/4}X$ with X being the streamwise triple-deck scale, $\lambda = 0.3320$,
and the conditions in equations (2.3)–(2.4) correspond, respectively to no-slin wake

In the above, $x = \lambda^{-5/4} X$ with X being the streamwise triple-deck scale, $\lambda = 0.3320$, and the conditions in equations (2.3)–(2.4) correspond, respectively, to no-slip, wake and the conditions in equations (2.3) – (2.4) correspond, respectively, to no-slip, wake
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symmetry, condition of matching, and merging with the Blasius solution. The viscous
triple-deck problem is closed by the relation $symmetry,$ condition of matching, and merging ${\bf w}$ triple-deck problem is closed by the relation

by the relation
\n
$$
\frac{\partial P}{\partial x} = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{A''(\zeta)}{x - \zeta} d\zeta,
$$
\n(2.5)

between the pressure $P(x)$ and the displacement function $A(x)$. Also note that even between the pressure $P(x)$ and the displacement function $A(x)$. Also note that even
though the governing equation (2.2) is parabolic, the pressure-displacement interac-
tion law (2.5) makes the whole problem elliptic. between the pressure $P(x)$ and the displacement f
though the governing equation (2.2) is parabolic, t
tion law (2.5) makes the whole problem elliptic.
The analytical structure of the underlying pro ough the governing equation (2.2) is parabolic, the pressure-displacement interac-
on law (2.5) makes the whole problem elliptic.
The analytical structure of the underlying problem for the interaction region in
variety of

tion law (2.5) makes the whole problem elliptic.
The analytical structure of the underlying problem for the interaction region in
a variety of physical situations leads, almost invariably, to the same fundamental
equati The analytical structure of the underlying problem for the interaction region in
a variety of physical situations leads, almost invariably, to the same fundamental
equation (2.2) with only a small change necessary in th a variety of physical situations leads, almost invariably, to the same fundamental
equation (2.2) with only a small change necessary in the boundary conditions. Com-
putations in this paper have been performed with the sha equation (2.2) with only a small change n
putations in this paper have been perform
the form of the trailing edge given by

ge given by
\n
$$
H(x) = \begin{cases}\n-\beta x & \text{for } x < 0, \\
0 & \text{for } x > 0.\n\end{cases}
$$
\n(2.6)

The large asymptotic behaviour of the displacement function, as well as the pres-The large asymptotic behaviour of the displacement function, as well as the pressure far upstream and downstream of the aerofoil, can be derived as in Sychev *et al.* (1998), and they are The large asymptotic
sure far upstream and c
(1998), and they are (1998) , and they are), $x \to -\infty$, $A(x) \to \gamma x^{1/3}$, ; $x \to \infty$, (2.7)

$$
A(x) \to \beta x + O(x^{-1/3}), \quad x \to -\infty, \qquad A(x) \to \gamma x^{1/3}, \quad x \to \infty, \tag{2.7}
$$

$$
P(x) \to -(\beta/\pi)\ln(x), \quad x \to \pm \infty, \tag{2.8}
$$

$$
P(x) \to -(\beta/\pi)\ln(x), \quad x \to \pm\infty,
$$
\n(2.8)

 $P(x) \rightarrow -(\beta/\pi) \ln(x), \quad x \rightarrow \pm \infty,$ (2.8)
where $\gamma = 0.89$ and β is the scaled wedge angle, which is connected to the physically
small angle ϕ through $\beta = Re^{1/4} \lambda^{-1/2} \phi$. where $\gamma = 0.89$ and β is the scaled wedge angle, which is connected to the physically
small angle ϕ through $\beta = Re^{1/4}\lambda^{-1/2}\phi$.
Numerical solutions were also obtained for the triple-deck flow over humps and
near a small angle ϕ through $\beta = Re^{1/4}\lambda^{-1/2}\phi$.
Numerical solutions were also obtained for the triple-deck flow over humps and

near a concave corner, for which the shape function $H(x)$ is given by

$$
H(x) = \begin{cases} 0 & \text{for } x < 0, \\ \beta x & \text{for } x > 0. \end{cases}
$$
 (2.9)

The hump shape is defined as in Smith $\&$ Bodonyi (1985):

defined as in Smith & Bodonyi (1985):
\n
$$
H(x) = \begin{cases} h(1-x^2)^2 & \text{for } -1 < x < 1, \\ 0 & \text{elsewhere.} \end{cases}
$$
\n(2.10)

 $T(x) = \begin{cases} 0 & \text{elsewhere.} \end{cases}$
The boundary conditions (2.3) and the far-downstream asymptotes (2.7), (2.8) for the flow over a corner and hump are also suitably modified and are given for example The boundary conditions (2.3) and the far-downstream asymptotes (2.7), (2.8) for the flow over a corner and hump are also suitably modified, and are given, for example, in Smith & Merkin (1982) In (2.9) and (2.10) β is The boundary conditions (2.3) and the far-downstream asymptotes (2.7), (2.8) for the flow over a corner and hump are also suitably modified, and are given, for example, in Smith & Merkin (1982). In (2.9) and (2.10), β flow over a corner and hump ε
in Smith & Merkin (1982). In
scaled hump size parameter. (*c*) *Numerical method of solution*

 (c) *Numerical method of solution*
For the numerical treatment of the trailing-edge flow problem just described, we
an the infinite physical region onto a finite computational domain by the transform For the numerical treatment of the trailing-edge flow problem just described, we
map the infinite physical region onto a finite computational domain by the transform
 $x = \tan(Z)$ so that the constraints far unstream and far d For the numerical treatment of the trailing-edge flow problem just described, we map the infinite physical region onto a finite computational domain by the transform $x = \tan(Z)$, so that the constraints far upstream and far map the infinite physica
 $x = \tan(Z)$, so that the

exactly at $Z = \pm \frac{1}{2}\pi$, 1π ysical region onto a finite computational domain by the transform
t the constraints far upstream and far downstream could be set
 $\frac{1}{2}\pi$, where $|x| \to \infty$, and the irregular behaviours are treated

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adequately in equation (2.7). In order to avoid the growing properties of the thickness
and the pressure in (2.7) and (2.8), we introduce the following transformations: adequately in equation (2.7) . In order to avoid the growing properties of the thickness and the pressure in (2.7) and (2.8) , we introduce the following transformations:

$$
A = \frac{\beta}{\pi} [x(\frac{1}{2}\pi - Z)] + \frac{2\gamma}{3^{1/2}} (1 + x^2)^{1/6} \sin(\frac{1}{6}(5\pi - 2Z)) + A_s,
$$

\n
$$
P = -\frac{\beta}{2\pi} \ln(1 + x^2) + \frac{\beta}{\pi(1 + x^2)} - \frac{2\gamma}{3^{3/2}} (1 + x^2)^{-1/3} \cos(\frac{1}{3}(\pi + 2Z)) + P_s.
$$
\n(2.11)

This idea was originally introduced in the triple-deck study of Smith & Merkin This idea was originally introduced in the triple-deck study of Smith & Merkin (1982). Note that with these transformations, P_s and $-dA_s/dx$ remain conjugate pairs and the inviscid interaction law then becomes This idea was originally introduced in the triple-dec (1982). Note that with these transformations, P_s and pairs and the inviscid interaction law then becomes pairs and the inviscid interaction law then becomes

$$
P_{\rm s} = \frac{1}{\pi} \int_{-\pi/2}^{\pi/2} \frac{A'_{\rm s}(\zeta)}{\tan(Z) - \tan(\zeta)} d\zeta.
$$
 (2.12)

 $P_s = \frac{1}{\pi} \int_{-\pi/2}^{\pi} \frac{\tan(Z) - \tan(\zeta)}{\tan(Z) - \tan(\zeta)} d\zeta.$ (2.12)
Working with A_s instead of A ensures that A_s decays asymptotically as $Z \to \pm \frac{1}{2}\pi$,
so that the integral equation (2.12) may be handled in a more system 1_{π} Working with A_s instead of A ensures that A_s decays asymptotically as $Z \to \pm \frac{1}{2}\pi$,
so that the integral equation (2.12) may be handled in a more systematic manner. In
this way, the elliptic inviscid law is treated Working with A_s instead of A ensures that A_s decays asymptotically as $Z \to \pm \frac{1}{2}\pi$, so that the integral equation (2.12) may be handled in a more systematic manner. In this way, the elliptic inviscid law is treated so that the integral equation (2.12) may be handled in a more systematic manner. In this way, the elliptic inviscid law is treated with a method introduced by Veldman (1979) in which the integration (2.12) is carried out at N local points leading to

$$
P_{\rm s}(x_j) = \sum_{i=0}^{N} \beta_{ij} A_{\rm si},
$$

where

$$
i=0
$$

$$
\beta_{ij} = \begin{cases} -f_{1j}, & i = 0, \\ f_{i-1j} - f_{ij}, & 1 \leq i \leq N-1, \\ f_{N-1j}, & i = N, \end{cases}
$$

 $\begin{cases} f_{N-1j}, & i=N, \end{cases}$ and $f_{ij} = [\tan(x_j) - \tan(x_{i+(1/2)})]^{-1}$. We choose this kind of approximation to the integral owing to the fact that it handles the singularity effectively by using implicit. $\binom{JN-1}{j}$, and $f_{ij} = [\tan(x_j) - \tan(x_{i+(1/2)})]^{-1}$. We choose this kind of approximation to the integral owing to the fact that it handles the singularity effectively by using implicit approximation of the interaction conditi and $f_{ij} = [\tan(x_j) - \tan(x_{i+(1/2)})]^{-1}$. We choose this kind of approximation to the integral owing to the fact that it handles the singularity effectively by using implicit approximation of the interaction condition (2.12), and integral owing to the fact that it handles the singularity effectively by using implicit approximation of the interaction condition (2.12) , and it remains superior to the other known methods, the so-called inverse or se approximation of the interaction condition (2.12) , and it remains superior to the other known methods, the so-called inverse or semi-inverse iterative methods, as far as the convergence rate is concerned.

The nonlinear interaction equation (2.2) is first differentiated with respect to Y to eliminate the pressure gradient. To complete the system we need one more equation, The nonlinear interaction equation (2.2) is first differentiated with respect to Y to eliminate the pressure gradient. To complete the system we need one more equation, which may be obtained from equation (2.2) by setting eliminate the pressure gradient
which may be obtained from e
flow, for instance, we obtain:

$$
P_x = \begin{cases} \psi'''(0), & x \le 0, \\ \psi'''(0) - \psi'(0)\psi'_x(0), & x > 0. \end{cases}
$$
 (2.13)

Equations $(2.2)-(2.13)$ are discretized using a Chebyshev collocation method (see, for example, Canuto *et al.* 1988) in the Y-direction, after truncating the semi-infinite Equations (2.2)–(2.13) are discretized using a Chebyshev collocation method (see,
for example, Canuto *et al.* 1988) in the \bar{Y} -direction, after truncating the semi-infinite
physical domain at a value \bar{Y}_{max} and for example, Canuto *et al.* 1988) in the Y-direction, after truncating the semi-infinite
physical domain at a value \bar{Y}_{max} and mapping onto the $[-1, 1]$ Chebyshev compu-
tational domain by means of a linear transfo physical domain at a value \bar{Y}_{max} and mapping onto the $[-1, 1]$ Chebyshev computational domain by means of a linear transformation. Standard second-order finite differences are used for the x-derivatives. This yield tational domain by means of a linear transformation. Standard second-order finite differences are used for the x -derivatives. This yields a nonlinear system, which is then linearized with a suitable initial guess using differences are used for the x-derivatives. This yields a nonlinear system, which is bined in a generalized matrix form, which is then solved using LU decomposition.

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2. Corner-flow results are shown for a various values of β . (a) Skin friction alors surface of the plate, (b) pressure distribution, and (c) displacement function. surface of the plate, (b) pressure distribution, and (c) displacement function.
3. Mean-flow results

The numerical results obtained by solving the fundamental triple-deck equations are presented in this section. Typically, 200 streamwise locations in the nite interval $\left[-\frac{1}{2}\pi,\right]$ e numerical results obtained by solving the fundamental triple-deck equations are
sented in this section. Typically, 200 streamwise locations in the finite interval
 $\frac{1}{2}\pi$, $\frac{1}{2}\pi$] and 64 Chebyshev collocation poi boundary \bar{Y}_{max} was chosen to be 10 for all the calculations. To check on the effect of
the computational grid, these parameters were halved or doubled accordingly for each
flow calculation. The code was first verif m this section. Typically, 200 streamwise locations in the finite interval
and 64 Chebyshev collocation points were used. The edge of the outer
 $\int_{\text{max}}^{\text{max}}$ was chosen to be 10 for all the calculations. To check on th $[-\frac{1}{2}\pi, \frac{1}{2}\pi]$ and 64 Chebyshev collocation points were used. The edge of the outer
boundary \bar{Y}_{max} was chosen to be 10 for all the calculations. To check on the effect of
the computational grid, these parame the computational grid, these parameters were halved or doubled accordingly for each flow calculation. The code was first verified by computing results for the triple-deck flow over a corner and also the triple-deck flow o w calculation. The code was first verified by computing results for the triple-deck
w over a corner and also the triple-deck flow over a hump.
A numerical solution of the nonlinear interaction problem for both cases has a

flow over a corner and also the triple-deck flow over a hump.
A numerical solution of the nonlinear interaction problem for both cases has also
been obtained by Ruban (1976, 1977). Smith & Merkin (1982) extended the calcu A numerical solution of the nonlinear interaction problem for both cases has
been obtained by Ruban (1976, 1977). Smith & Merkin (1982) extended the c
lations for larger values of β and h. Our results are shown in figu been obtained by Ruban (1976, 1977). Smith & Merkin (1982) extended the calculations for larger values of β and h . Our results are shown in figures 2 and 3. In general, good agreement is found with the results of Rub

lations for larger values of β and h . Our results are shown in figures 2 and 3.
In general, good agreement is found with the results of Ruban (1976, 1977), Smith & Merkin (1982) and Smith & Bodonyi (1985). However, R In general, good agreement is found with the results of Ruban (1976, 1977), Smith & Merkin (1982) and Smith & Bodonyi (1985). However, Ruban (1976) concluded that in the case of a concave corner, flow separation first app & Merkin (1982) and Smith & Bodonyi (1985). However, Ruban (1976) concluded that in the case of a concave corner, flow separation first appears when $\beta = 2$ (see also Sychev *et al.* 1998). Yet from the calculations of Sm *Phil. Trans. R. Soc. Lond.* A (2000)

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Results for the flow over the hump defined by (2.10) are shown for a various v
h. (a) Pressure distribution, (b) displacement function, and (c) skin friction.

h. (a) Pressure distribution, (b) displacement function, and (c) skin friction.
fig. 5 in their paper), separation is found to occur at a larger value of $\beta = 2.51$. From fig. 5 in their paper), separation is found to occur at a larger value of $\beta = 2.51$. From
our calculations we find that separation is encountered around $\beta = 2.09$, which is
relatively close to the value found by Buban fig. 5 in their paper), separation is found to occ
our calculations we find that separation is enc
relatively close to the value found by Ruban. relatively close to the value found by Ruban.
(a) *Wedged trailing-edge results*

(a) Wedged trailing-edge results
Calculations were next carried out for the wedged trailing-edge flow. Figure 4
ows the distribution of the skin friction and centreline velocity along the wake axis Calculations were next carried out for the wedged trailing-edge flow. Figure 4
shows the distribution of the skin friction and centreline velocity along the wake axis
of symmetry for a range of values of scaled wedge angl Calculations were next carried out for the wedged trailing-edge flow. Figure 4 shows the distribution of the skin friction and centreline velocity along the wake axis of symmetry for a range of values of scaled wedge angl shows the distribution of the skin friction and centreline velocity along the wake axis of symmetry for a range of values of scaled wedge angle β between 0 and 3.8. As β increases, separation occurs and a recirculat of symmetry for a range of values of scaled wedge angle β between 0 and 3.8. As β increases, separation occurs and a recirculating zone of fluid particles appears in the vicinity of the trailing edge, the extent of increases, separation occurs and a recirculating zone of fluid particles appears in the vicinity of the trailing edge, the extent of which increases with increasing β . From figure 5a the critical parameter leading to a vicinity of the trailing edge, the extent of which increases with increasing β . From figure 5*a* the critical parameter leading to a change from attached to separated flow appears, from our calculation, to be 2.56, whi figure 5a the critical parameter leading to a change from attached to separated flow
appears, from our calculation, to be 2.56, which is in quite good agreement with the
value of 2.6 obtained by Ruban (1977). Smith & Merk appears, from our calculation, to be 2.56, which is in quite good agreement with the value of 2.6 obtained by Ruban (1977). Smith & Merkin (1982) found that separation first occurs for $\beta = 2.38$ for the same reduced shap value of 2.6 obtained by Ruban (1977). Smith & Merkin (1982) found that separation
first occurs for $\beta = 2.38$ for the same reduced shape (see fig. 7 in their paper). The
corresponding dependence of the distribution on th corresponding dependence of the distribution on the pressure as well as the displacement thickness on x for given values of β is shown in parts (b) and (c) of figure 5.

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Overall, these results agree well with the published results of Jobe $\&$ Burggraf (1974) Overall, these results agree well with the published results of Jobe & Burggraf (1974) and Ruban (1977), but differ from Smith & Merkin (1982), particularly for values of β close to separation. It is noted that our met Overall, these results agree well with the published results of Jobe & Burggraf (1974) and Ruban (1977), but differ from Smith & Merkin (1982), particularly for values of β close to separation. It is noted that our met and Ruban (1977), but differ from Smith & Merkin (1982), particularly for values of β close to separation. It is noted that our method of solution is different from those of Ruban (1976, 1977) and Smith & Merkin (1982) β close to separation. It is noted that our method of solution is different from those of
Ruban (1976, 1977) and Smith & Merkin (1982). In our method we used Chebyshev
collocation to approximate the vertical derivative Ruban (1976, 1977) and Smith & Merkin (1982). In our method we used Chebyshev collocation to approximate the vertical derivatives instead of finite differencing used
in those latter papers. We believe that the differences collocation to approximate the vertical derivatives instead of finite differencing used
in those latter papers. We believe that the differences between the results of Smith
& Merkin (1982) and our results arise from error in those latter papers. We believe that the differences between the results of Smith $\&$ Merkin (1982) and our results arise from errors introduced in the transformed wake symmetry boundary conditions that Smith $\&$ Mer calculations. An increase in the parameter β leads to the pressure gradient becoming adversed the separated flow region is pushed upstream ahead of the trailing edge. Although

calculations.
An increase in the parameter β leads to the pressure gradient becoming adverse
and the separated flow region is pushed upstream ahead of the trailing edge. Although
the recirculating region gets widened, An increase in the parameter β leads to the pressure gradient becoming adverse
and the separated flow region is pushed upstream ahead of the trailing edge. Although
the recirculating region gets widened, the fluid part and the separated flow region is pushed upstream ahead of the trailing edge. Although the recirculating region gets widened, the fluid particles here possess a comparably small value of velocity. We were only able to comp

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Figure 5. Wedged trailing-edge flow results are shown for a range of values of β . (*a*) Variation
of the minimum value of the skin friction with respect to β , (*b*) pressure distribution (*c*) Figure 5. Wedged trailing-edge flow results are shown for a range of values of β . (*a*) Variation of the minimum value of the skin friction with respect to β , (*b*) pressure distribution, (*c*) displacement function of the minimum value of the skin friction with respect to β , (b) pressure distribution, (c) displacement function.

after which our numerical procedure failed. As seen from $figure\ 5a$, the minimum stead in the numerical procedure failed. As seen from figure 5*a*, the minimum skin friction reaches a minimum and then shows a tendency to increase slightly as β approaches 3.8. This sort of phenomenon is also encount after which our numerical procedure failed. As seen from figure 5*a*, the minimum
skin friction reaches a minimum and then shows a tendency to increase slightly
as β approaches 3.8. This sort of phenomenon is also enco skin friction reaches a minimum and then shows a tendency to increase slightly
as β approaches 3.8. This sort of phenomenon is also encountered in the work of
Korolev (1991, 1992) for the corner flow and he explains th as β approaches 3.8. This sort of phenomenon is also encountered in the work of Korolev (1991, 1992) for the corner flow and he explains this behaviour as hysteresis, implying non-existence of the solutions beyond this Korolev (1991, 1992) for the corner flow and he explains this behaviour as hysteresis,
implying non-existence of the solutions beyond this critical value. It also implies that
the solution of the interaction problem may e implying non-existence of the solutions beyond this critical value. It also implies that
the solution of the interaction problem may exist only within a certain range of the
angle parameter β , and multiple solutions of the solution of the interaction problem may exist only within a certain range of the angle parameter β , and multiple solutions of the problem beyond this range may be possible. It would be useful to extend these calcul angle parameter β , and multiple solutions of the problem beyond this range may be possible. It would be useful to extend these calculation using more direct methods (as in Korolev (1992)) and study possible hysteresis (as in Korolev (1992)) and study possible hysteresis effects of the separated flow.
(*b*) *Composite solutions*

Having solved the triple-deck equations, it is now an easy task to construct asymptotic composite solutions uniformly valid in the triple-deck region. Due to small contributions from the upper-deck expansion, only the main-deck and lower-deck solutions will be considered in the formation of a composite solution. It will, however, be Reynolds number dependent, and here we suppose that the Reynolds number is tions will be considered in the formation of a composite solution. It will, however,
be Reynolds number dependent, and here we suppose that the Reynolds number is
measured based on the main chord-line length of the aerofo be Reynolds number dependent, and here we suppose that the Reynolds number is
measured based on the main chord-line length of the aerofoil. Let U_m denote the
main-deck solution given by (2.1), U_1 the lower-deck solut measured based on the main chord-line length of the aerofoil. Let U_m denote the main-deck solution given by (2.1), U_1 the lower-deck solution to be obtained from (2.2)–(2.5), and U_{mtc} the matching between them. main-deck solution given by (2.1) , U_1 t
 (2.2) – (2.5) , and U_{mtc} the matching bets
layer solution with (2.1) we find that layer solution with (2.1) we find that $(A(X) + H(X)))$ as $Y \to \infty$. (3.1)

$$
U_{\text{mtc}}(Y) = \lambda (Y + Re^{-1/8}(A(X) + H(X))) \quad \text{as } Y \to \infty. \tag{3.1}
$$

 $U_{\text{mtc}}(Y) = \lambda (Y + Re^{-1/8}(A(X) + H(X)))$ as $Y \to \infty$. (3.1)
Taking into account that $\overline{Y} = Re^{1/8}Y$, where \overline{Y} is the lower-deck boundary-layer
scale we obtain the solution for the composite expansion as Taking into account that $\overline{Y} = Re^{1/8}Y$, where \overline{Y} is the lower-scale, we obtain the solution for the composite expansion as scale, we obtain the solution for the composite expansion as

$$
U(Y) = U_{\mathcal{B}}(Y) + Re^{-1/8}(A(X) + H(X))U'_{\mathcal{B}}(Y) + Re^{-1/8}U_{1}(Re^{1/8}Y) - \lambda(Y + Re^{-1/8}(A(X) + H(X))).
$$
 (3.2)

 $-\lambda(Y + Re^{-1/8}(A(X) + H(X))).$ (3.2)
Notice that this solution satisfies the matching requirement used in the boundary
condition (2.4) as well as $U(Y = 0) = 0$ and $U(Y \to \infty) \to 1$ condition (2.4), as well as $U(Y = 0) = 0$ and $U(Y \to \infty) \to 1$.
Sample composite mean velocity profiles obtained from equation (3.2) for $Re = 500$ obtice that this solution satisfies the matching requirement used in the boundary
ndition (2.4), as well as $U(Y = 0) = 0$ and $U(Y \to \infty) \to 1$.
Sample composite mean velocity profiles obtained from equation (3.2) for $Re = 500$

condition (2.4), as well as $U(Y = 0) = 0$ and $U(Y \to \infty) \to 1$.
Sample composite mean velocity profiles obtained from equation (3.2) for $Re = 500$
are shown in figure 6 for the wedged trailing-edge flow. Each graph here corres Sample composite mean velocity profiles obtained from equation (3.2) for $Re = 500$ are shown in figure 6 for the wedged trailing-edge flow. Each graph here corresponds to $\alpha_{\star} = 0, 0.5, 1.5$ and 2, respectively, where $\$ to $\alpha_{\star} = 0, 0.5, 1.5$ and 2, respectively, where $\alpha_{\star} = \sqrt{\lambda \beta}$. It is seen that these profiles correspond to the Gaussian type of wake profiles far downstream. One can also to $\alpha_{\star} = 0, 0.5, 1.5$ and 2, respectively, where $\alpha_{\star} = \sqrt{\lambda \beta}$. It is seen that these profiles correspond to the Gaussian type of wake profiles far downstream. One can also see from the third portion of figure 6 tha correspond to the Gaussian type of wake profiles far downstream. One can also
see from the third portion of figure 6 that the flow separation has already started at
 $\alpha_{\star} = 1.5$. Therefore, the triple-deck velocity profi see from the third portion of figure 6 that the flow separation has already started at $\alpha_{\star} = 1.5$. Therefore, the triple-deck velocity profiles obtained here have the advantage that the linear stability properties may $\alpha_{\star} = 1.5$. Therefore, the triple-deck velocity profiles obtained here have the advantage that the linear stability properties may be investigated even for the regions of flow reversal. This is discussed in the next se

4. Absolute instability results

4. Absolute instability results
Smith & Bodonyi (1985) have shown that the triple-deck profiles in the boundary-
layer flow past an obstacle on a surface are inviscidly unstable. Because the triple-Smith & Bodonyi (1985) have shown that the triple-deck profiles in the boundary-
layer flow past an obstacle on a surface are inviscidly unstable. Because the triple-
deck solutions in the separation zone for this kind of Smith & Bodonyi (1985) have shown that the triple-deck profiles in the boundary-
layer flow past an obstacle on a surface are inviscidly unstable. Because the triple-
deck solutions in the separation zone for this kind of layer flow past an obstacle on a surface are inviscidly unstable. Because the triple-
deck solutions in the separation zone for this kind of flow give rise to inflectional
profiles, they were able to compute the inviscid i deck solutions in the separation zone for this kind of flow give rise to inflectional
profiles, they were able to compute the inviscid instability properties, on the grounds
that the lower-deck modes have spatial or tempor that the lower-deck modes have spatial or temporal growth rates much higher than those associated with the main- or upper-decks, due to the inflectional velocity prothat the lower-deck modes have spatial or temporal growth rates much higher than
those associated with the main- or upper-decks, due to the inflectional velocity pro-
files having the shortest scale in physical variables those associated with the main- or upper-decks, due to the inflectional velocity pro-
files having the shortest scale in physical variables x and y (both $O(Re^{-5/8})$. This
leads to the inviscid Rayleigh equation (see Smith files having the shortest scale in physical variables x and y (both $O(Re^{-5/8})$. This
leads to the inviscid Rayleigh equation (see Smith & Bodonyi 1985). The nonlinear
results presented in Duck (1985, 1988) indicated that leads to the inviscid Rayleigh equation (see Smith & Bodonyi 1985). The nonlinear
results presented in Duck (1985, 1988) indicated that rapid growth of the spectral
solution occurs for large wavenumbers, suggesting the de results presented in Duck (1985, 1988) indicated that rapid growth of the spectral
solution occurs for large wavenumbers, suggesting the development of a short-scale
Rayleigh-type instability. Taking this into account, we solution occurs for large wavenumbers, suggesting the development of a short-scale
Rayleigh-type instability. Taking this into account, we first attempted to investigate
whether the wedged trailing-edge flow also has infle Rayleigh-type instability. Taking this into account, we first attempted to investigate
whether the wedged trailing-edge flow also has inflectional lower-deck velocity pro-
files. However, these profiles do not appear to ha whether the wedged trailing-edge flow also has inflectional lower-deck velocity pro-
files. However, these profiles do not appear to have any inflection points. This may be
solely because acceleration of the fluid particle files. However, these profiles do not appear to have any inflection points. This may be solely because acceleration of the fluid particles is possible in the trailing-edge flow, whereas acceleration of the fluid particles over the obstacle, followed by deceleration, seemingly raises the occurrence of inflecti whereas acceleration of the fluid particles over the obstacle, followed by deceleration, seemingly raises the occurrence of inflectional points in the boundary-layer flow past wall distortions.
The presence of inflectional seemingly raises the occurrence of inflectional points in the boundary-layer flow past

bility may also exist. The lower-deck velocity profiles for the flow over the hump

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Figure 6. Composite velocity profiles of the wedged trailing-edge flow are shown for $Re = 500$.
Curves correspond to the x locations (from left to right) -32.0 , 0.01, 0.5, 1.0, 1.5, 3.1 and 5.2, respectively.

shape (2.10) were investigated using our inviscid Rayleigh equation solver. In figure 7^a the presence of a saddle-point-type phenomenon is demonstrated. Figure 7^b shows that the saddle point in figure 7a is a pinch point with $\omega_i > 0$, suggesting ure 7*a* the presence of a saddle-point-type phenomenon is demonstrated. Figure 7*b* shows that the saddle point in figure 7*a* is a pinch point with $\omega_i > 0$, suggesting absolute instability. Here $\omega = \omega_r + i\omega_i$, $\alpha = \alpha_r +$ shows that the saddle point in figure 7*a* is a pinch point with $\omega_i > 0$, suggesting absolute instability. Here $\omega = \omega_r + i\omega_i$, $\alpha = \alpha_r + i\alpha_i$ are the scaled complex frequency and wavenumber of the disturbances, respective absolute instability. Here $\omega = \omega_r + i\omega_i$, $\alpha = \alpha_r + i\alpha_i$
and wavenumber of the disturbances, respectively.
given in the α, ω -planes for different hump sizes.
According to figure 7c, the instability starts al and wavenumber of the disturbances, respectively. In figure 7c, d, branch points are given in the α, ω -planes for different hump sizes.
According to figure 7c, the instability starts almost at the peak point for each

hump size and extends far downstream of the hump. Increasing the disturbance According to figure 7c, the instability starts almost at the peak point for each
hump size and extends far downstream of the hump. Increasing the disturbance
height also causes an increase in the absolute growth rate of th hump size and extends far downstream of the hump. Increasing the disturbance
height also causes an increase in the absolute growth rate of the disturbances, as
might be expected physically. We suggest, therefore, that the height also causes an increase in the absolute growth rate of the disturbances, as
might be expected physically. We suggest, therefore, that the instability observed by
Duck (1985, 1988) leading to a finite-time breakdown might be expected physically. We suggest, therefore, that the instabilit
Duck (1985, 1988) leading to a finite-time breakdown of the lower-de-
layer solution may be related to the absolute instability found here.
The flow Ick (1985, 1988) leading to a finite-time breakdown of the lower-deck boundary-
The flow in the vicinity of the trailing edge changes most rapidly, and, therefore, it
joht be expected that the viscous triple-deck region cl

layer solution may be related to the absolute instability found here.
The flow in the vicinity of the trailing edge changes most rapidly, and, therefore, it
might be expected that the viscous triple-deck region close to th The flow in the vicinity of the trailing edge changes most rapidly, and, therefore, it
might be expected that the viscous triple-deck region close to the trailing edge would
exhibit a strong influence on the stability prop might be expected that the viscous triple-deck region close to the trailing edge would exhibit a strong influence on the stability properties. For this reason the composite basic velocity profiles obtained from the triplebasic velocity profiles obtained from the triple-deck mean flow are considered next.
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 ω -planes, for hump height $h = 2$ and x station $x = 0.94$. Pinching occurs at $\alpha = (0.21, -0.08)$ and $\omega = (0.8, 0.07)$. (+) and (-) show, respectively, increasing and decreasing ω_i . Branch points are demonstrated ((c) and (d)) along the triple-deck region in the α - and ω -planes for hump sizes $h = 2, 2.5, 3$ a are demonstrated ((c) and (d)) along the triple-deck region in the α - and ω -planes for hump

As displayed in figure 6, the composite velocity profiles apparently have inflectional As displayed in figure 6, the composite velocity profiles apparently have inflectional
points somewhere in the middle of the boundary layer. This motivated us to explore
the inviscid instability characteristics of such pro As displayed in figure 6, the composite velocity profiles apparently have inflectional
points somewhere in the middle of the boundary layer. This motivated us to explore
the inviscid instability characteristics of such pro points somewhere in the middle of the boundary layer. This motivated us to explore
the inviscid instability characteristics of such profiles. These profiles were fed into the
inviscid Rayleigh solver and afterwards branch the inviscid instability characteristics of such profiles. These profiles were fed into the
inviscid Rayleigh solver and afterwards branch points were located. An initial search
for branch points on the wedge ahead of the inviscid Rayleigh solver and afterwards branch points were located. An initial search
for branch points on the wedge ahead of the trailing edge did not yield any absolutely
unstable branch points, suggesting that the flow for branch points on the wedge ahead of the trailing edge did not yield any absolutely
unstable branch points, suggesting that the flow on the wedge is only convectively
unstable. Figure 8 shows branch points that satisfy unstable branch points, suggesting that the flow on the wedge is only convectively
unstable. Figure 8 shows branch points that satisfy the Briggs–Bers criterion for the
flow in the wake. (In the wake region there is a chan unstable. Figure 8 shows branch points that satisfy the Briggs–Bers criterion for the flow in the wake. (In the wake region there is a change in the boundary conditions and results are shown for disturbances that satisfy flow in the wake. (In the wake region there is a change in the boundary conditions
and results are shown for disturbances that satisfy the condition of zero pressure on
the centreline.) The solid curves are for $Re = 500$, and results are shown for disturbances that satisfy the condition of zero pressure on
the centreline.) The solid curves are for $Re = 500$, the dashed curves for $Re = 1000$,
and the dotted ones for $Re = 10000$, plotted for se the centreline.) The solid curves are for $Re = 500$, the dashed curves for and the dotted ones for $Re = 10000$, plotted for several values of the p It is seen that for α_{\star} larger than 0.5 the wake is absolutely unstabl and the dotted ones for $Re = 10000$, plotted for several values of the parameter α_{\star} .
It is seen that for α_{\star} larger than 0.5 the wake is absolutely unstable.
We also observe from figure 8 that, for the zero pres

only convective instability $(\omega_i < 0)$, in contrast to the suggestions made by Woodley We also observe from figure 8 that, for the zero pressure gradient case, there is
only convective instability ($\omega_i < 0$), in contrast to the suggestions made by Woodley
& Peake (1997), who speculated that the double Blasi only convective instability ($\omega_i < 0$), in contrast to the suggestions made by Woodley & Peake (1997), who speculated that the double Blasius profile might be absolutely unstable. The effect of increasing the wedge angle *Phil. Trans. R. Soc. Lond.* A (2000)

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b $\begin{array}{ccc} 0 & 1 & 2 & x & 3 & 4 & 5 \end{array}$
Figure 8. Absolute instability range for the flow over a wedged trailing edge is shown along the triple-deck region for several values of parameter α . Solid lines denote $Re = 500$ d Figure 8. Absolute instability range for the flow over a wedged trailing edge is shown along the triple-deck region for several values of parameter α_{\star} . Solid lines denote $Re = 500$, dashed lines denote $Re = 10000$ triple-deck region for several values of parameter α_{\star} . Solid lines denote $Re = 500$, dashed lines denote $Re = 1000$, and dotted lines denote $Re = 10000$.

rate and to make the region of absolute instability much wider. This also implies that rate and to make the region of absolute instability much wider. This also implies that
separation ensures the existence of absolute instability by enhancing the instability
region at least for the underlying picture of the rate and to make the region of absolute instability much wider. This also implies that separation ensures the existence of absolute instability by enhancing the instability region, at least for the underlying picture of t separation ensures the existence of absolute instability by enhancing the instability region, at least for the underlying picture of the boundary-layer flow considered here.
Increasing the wedge angle parameter β is ak region, at least for the underlying picture of the boundary-layer flow considered here.
Increasing the wedge angle parameter β is akin to increasing the aerofoil thickness, which is likely to promote stronger separatio

5. Summary and conclusions

5. Summary and conclusions
In this paper we have considered both the solutions of the triple-deck equations and
the stability of the solutions for the flows over a wall irregularity a corner and a In this paper we have considered both the solutions of the triple-deck equations and
the stability of the solutions for the flows over a wall irregularity, a corner, and a
wedged trailing edge. The calculations show that In this paper we have considered both the solutions of the triple-deck equations and
the stability of the solutions for the flows over a wall irregularity, a corner, and a
wedged trailing edge. The calculations show that the stability of the solutions for the flows over a wall irregularity, a corner, and a wedged trailing edge. The calculations show that separation from the surface of the body begins, respectively, at $\beta = 2.09$ and 2.56 wedged trailing edge. The calculations show that separation from the surface of the body begins, respectively, at $\beta = 2.09$ and 2.56 for the corner and wedged trailing edge. These values have been found to compare well w body begins, respectively, at $\beta = 2.09$ and 2.56 for the corner and wedged trailing edge. These values have been found to compare well with the ones obtained by Ruban (1976, 1977). The maximum wedge angle for which we we edge. These values have been found to compare well with the ones obtained by Ruban (1976, 1977). The maximum wedge angle for which we were able to obtain results is $\beta = 3.8$. The work of Korolev (1991, 1992) would sugges Ruban (1976, 1977). The maximum wedge angle for which we were able to obtain results is $\beta = 3.8$. The work of Korolev (1991, 1992) would suggest that separation hysteresis may arise after this critical value. Our invisci results is $\beta = 3.8$. The work of Korolev (1991, 1992) would suggest that separation
hysteresis may arise after this critical value. Our inviscid stability results have shown
that absolute instability exists in the triple hysteresis may arise after this critical va
that absolute instability exists in the tri
region behind a wedged trailing edge.
Composite basic velocity profiles have that absolute instability exists in the triple-deck flow over the hump and in the wake
region behind a wedged trailing edge.
Composite basic velocity profiles have been constructed for a wedged trailing edge,

region behind a wedged trailing edge.
Composite basic velocity profiles have been constructed for a wedged trailing edge,
including some wedge angles leading to flow separation. The double Blasius profile
that was suggest Composite basic velocity profiles have been constructed for a wedged trailing edge,
including some wedge angles leading to flow separation. The double Blasius profile
that was suggested to be absolutely unstable by Woodle including some wedge angles leading to flow separation. The double Blasius profile
that was suggested to be absolutely unstable by Woodley $\&$ Peake (1997) has been
found to be convectively unstable in the triple-deck re that was suggested to be absolutely unstable by Woodley $\&$ Peake (1997) has been found to be convectively unstable in the triple-deck region. Other composite velocity profiles have been found to exhibit similar absolute profiles have been found to exhibit similar absolute instability characteristics, as

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that increasing the wedge angle creates a zone of separated flow which in turn ensures
a much larger extent of absolutely unstable flow only behind the trailing edge that increasing the wedge angle creates a zone of separated flow which in turn ensume a much larger extent of absolutely unstable flow only *behind* the trailing edge.
The current method can also be used to solve the tripl at increasing the wedge angle creates a zone of separated flow which in turn ensures
much larger extent of absolutely unstable flow only *behind* the trailing edge.
The current method can also be used to solve the triple-d

a much larger extent of absolutely unstable flow only *behind* the trailing edge.
The current method can also be used to solve the triple-deck equations for the supersonic corner and wedged trailing-edge flows. For the sup The current method can also be used to solve the triple-deck equations for the supersonic corner and wedged trailing-edge flows. For the supersonic flows, the problem reduces to an even simpler partial differential form, s supersonic corner and wedged trailing-edge flows. For the supersonic flows, the problem reduces to an even simpler partial differential form, since the global interaction
law is replaced by Ackeret's formula. Work on compu lem reduces to an even simpler partial differential form, since the global interaction
law is replaced by Ackeret's formula. Work on computing solutions for the supersonic
flow and for non-aligned aerofoils is currently i law is replaced
flow and for r
et al. 1999).

et al. 1999).
The authors have benefited from discussions with Professor Anatoly Ruban on his and the
current work The authors had
current work.

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